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SURVEYING



BY JASPAL SIR

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FUNDAMENTALS

→ Surveying is the Process of determining the Relative position of point on, above and below the Surface of other by means of direct and indirect measurement of "Distance", Direction, & Elevation

it may Also be termed As Process of Establishing the points by Predetermined, Angular and linear measurement.

— Due to the Curvature of the Earth the Surveying is Primarily Classified into 2

NOTE Earth is An oblate Spheroid [Here Spheroid is obtained by Rotating An Ellipse about its one of the Axes. Since the Polar dia of Earth is approximately 43.95 km Smaller than Equatorial dia. it is termed As oblate Spheroid.

(A) Plane Surveying (ii) Geodetic Surveying

(A) PLANE SURVEYING

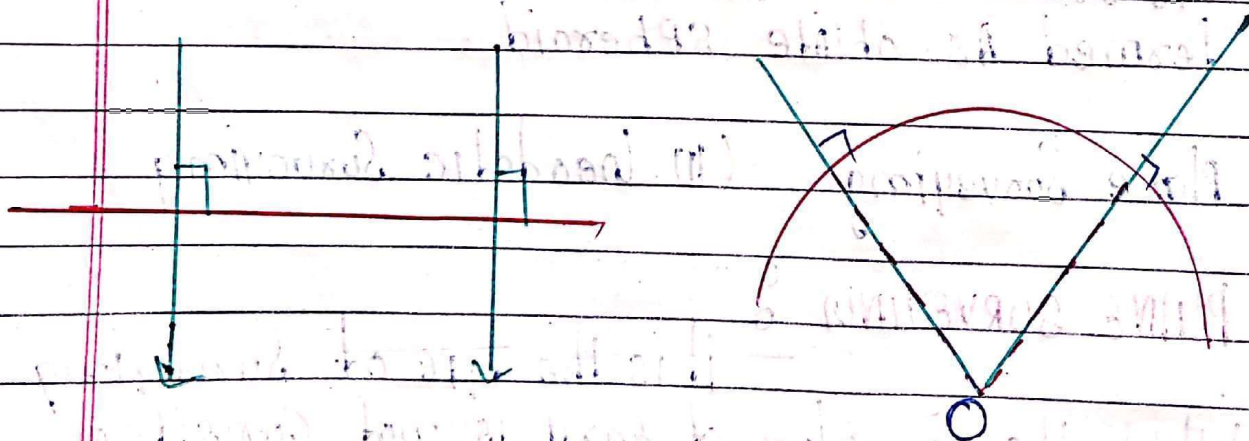
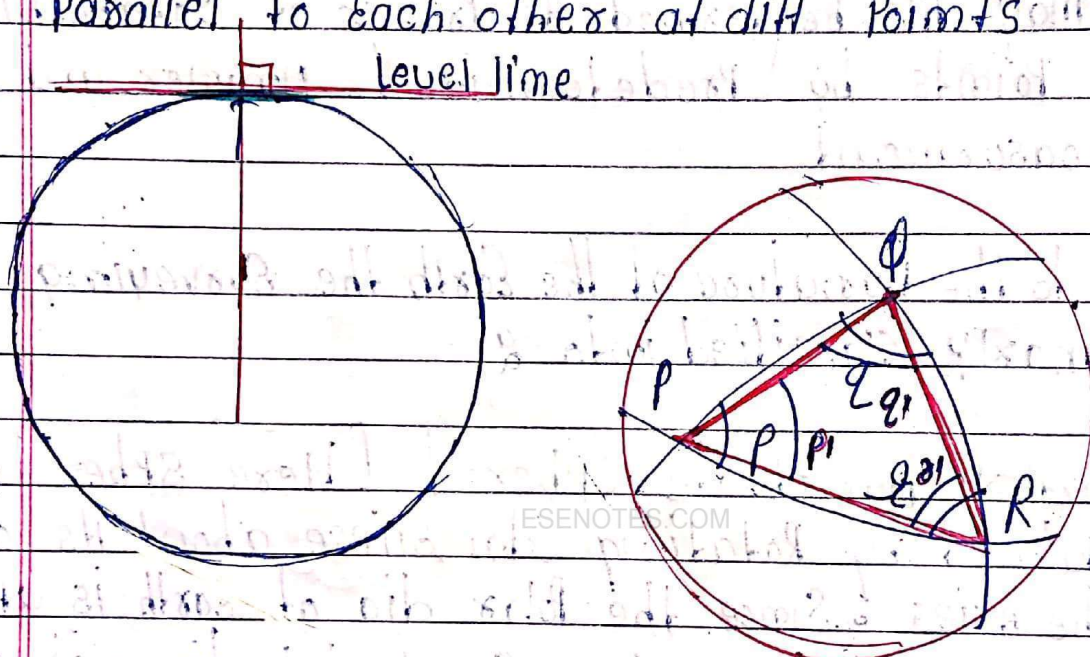
it is the type of Surveying in which the Curvature of Earth is not Consider or the mean Surface of the Earth is to be Planee

— it is Suitable to be used for Small Area of work

— it is Consider for Area < 250 Km²

→ All the lines and triangles formed during survey in this case will be considered as plane line and plane triangle.

→ Here level line is considered to be straight line & plumb lines (line represented by freely suspended "Plumb bob" are considered to be parallel to each other at diff. points).



9) GEODETIC SURVEYING

That type of surveying in which curvature or shape of Earth is taken into consideration. Here all lines lying in the surface are curved lines and triangles are spherical triangle.

- it is suitable to be used when Area of Surface is large (generally $> 250 \text{ km}^2$) and high degree of Precision is Required.
- This Surveying is done By department of national Survey of India to establish the Control Points which are taken as Reference Point for marking other points in plane Surveying.

Date.....

- Note: 1) For any triangle with area 195.5 km^2 , some of interior angle would be just $1''$ (1 sec) extra in geodetic survey.
- 2) For any line of length 12 km , it is just 1 cm extra in geodetic survey.

Surveying can also be classified as

(I) NATURE OF SURVEY

(a) LAND SURVEY

- Here surveying is done for the features of land.
- It is further classified into

(i) TOPOGRAPHICAL SURVEY'S

- This consists of horizontal and vertical location of certain points by linear and angular measurement to determine physical features like: River, Lake, Pond, Forest, hill [Natural], Road, Railway track, Canal, town, villages [Artificial].

(ii) CADASTRAL SURVEY'S

These survey are made for fixing the property line & calculation of land area or the transfer of land property.

(iii) CITY SURVEY

- These survey's are made in connection with the construction of streets, water pipe line, sewers, LPG line, optical fibre.

(B) MARINE / HYDROGRAPHIC SURVEY.

- It deals with water bodies for purpose of Navigation water supply, mean sea level, Reservoir functioning, work disposals.
- It consist of measurement of discharge of streams, stage of water body, & fluctuation in oceans tides.

(C) ASTRONOMICAL SURVEY

- It is carried out for determining the absolute location of any point on the surface of Earth with the help of heavenly / Celestial / special body.
E.g. SUN, any fixed star.

(II) BASED ON OBJECT OF SURVEY.

5

(A) ENGINEERING SURVEY

- This is done to find the sufficient data to be used for Engg. design works like roads, Reservoir, sewers, trench, foundation etc.

(B) MILITARY SURVEY

- It is used for determining the points of strategic importance

(C) GEOLOGICAL SURVEY

- It deals with determination of formation of Earth strata

(D) ARCHAEOLOGICAL SURVEY

- This is used to find the existence of Past civilisation

(E) MINE SURVEY :

- It is being carried out for Exploration of new mines.

(III) BASED ON THE BASIS OF INSTRUMENT USED

(i) CHAIN SURVEYING

- It is the simplest type of surveying in which only linear measurement are taken either with chain or tape.

Angular measurement are not taken in chain surveying.

(ii) COMPASS SURVEYING

- In this type of surveying horizontal angles are measured with the help of Magnetic Compass in addition to this linear measurement is taken with the help of chain or tape.
- Although magnetic Compass is not precise in measuring angles hence this surveying is not very accurate. However, it is more accurate than chain surveying.

(iii) LEVELLING

- In this type of survey levelling instrument is used to determine relative elevation (level) of various points in the vertical plane.

(iv) PLANE TABLE SURVEYING

- In this survey a "Map" or plan is prepared in field by observing the terrain after determining the direction of various lines and measuring the linear distance with chain or tape.
- The accuracy of this survey is low but in this case measurement and plotting are done simultaneously.

(V) THEODOLITE SURVEYING

- In this type of survey theodolite is used for measurement of horizontal & vertical angle
- Its accuracy is more as compared to Compass Survey
It is further classified as

TRANSVERSE SURVEY - In this various station forms a polygon	TRIANGULATION SURVEY - In this various station forms system of triangles.
--	--

(VI) TACHEOMETRIC SURVEY

- In this type of survey, a special type of theodolite called the tachometer is used which is fitted with cross hairs in addition to central horizontal hair.
- In this horizontal angle, horizontal distances and vertical distances are measured with tachometer.
- It is extremely easy & gives better result than theodolite survey in rough survey.

(VII) PHOTOGRAMMETRIC SURVEY.

- Photogrammetry is process of taking measurements with help of photograph.
- It is generally used for topographic mapping of large area.
- It is used for areas which are difficult to access
- Photographs are preferred to be taken from ground but if not possible aerial photography used.

(VIII) EDM [ELECTRONIC DISTANCE MEASUREMENT] SURVEYING

- In this type of survey linear measurement is done with the help of EDM, here also control point forms the

The triangle, hence termed as triangulation.

- Triangulation survey in which all length of all sides of triangle is measured is termed as "TRIANGULATION"

PRINCIPAL OF SURVEYING

The fundamental principal of surveying are as follow

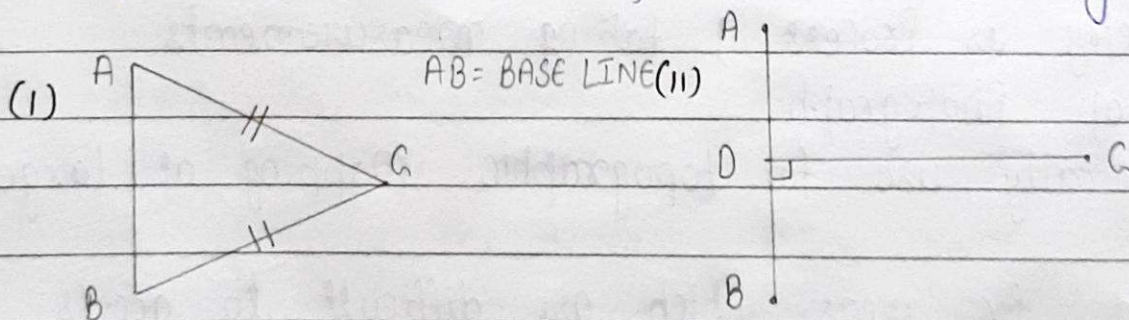
(A) WORKING FROM WHOLE TO PART

- whether plane or geodetic survey is done, work is to be carried out whole to part.
- In this we establish system of control point with high degree of accuracy.
- Minor control points are then established by less precise method
- By doing so, accumulation of the error is avoided and minor errors are localised

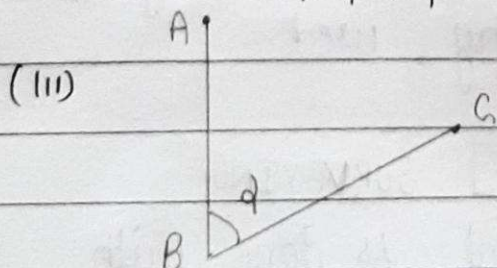
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(B) LOCATION OF A POINT BY MEASUREMENT FROM TWO POINTS

The relative position of the points to be surveyed should be located by measurement from atleast two points of reference, the position of which is already fixed

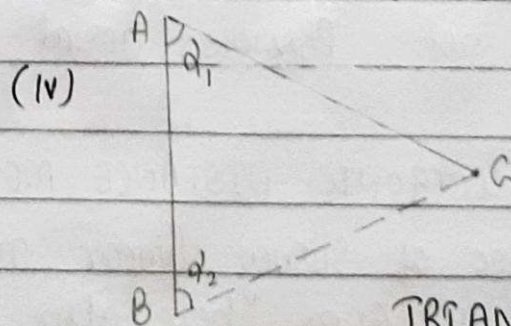


CHAIN SURVEYING

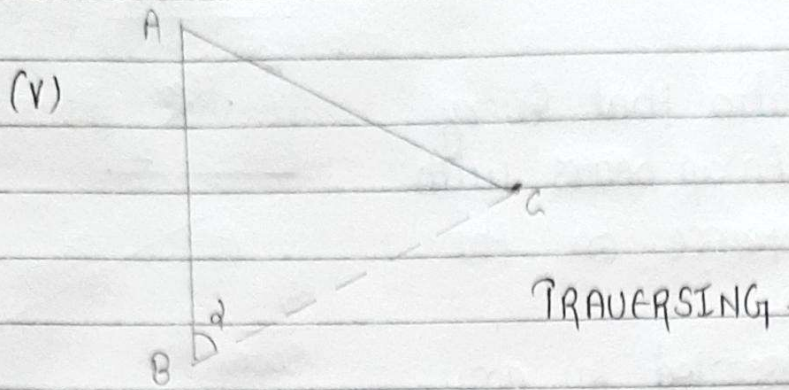


TRAVERSING

OFFSET METHOD



TRIANGULATION



Note: In second, (iii), (iv) Case the concerned point "c" can be in vertical plane also

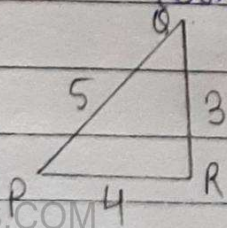
PLAN AND MAP

- A Plan is a graphical representation to some scale, of the features on, near or below surface of Earth.
- Since, the surface of the Earth is curved and the paper of the ~~plane~~ plan is plane, no part of the surface can be represented on such maps without distortion.
- Hence it is suitable for plane surveying in which area involved is comparatively smaller.
- The representation is called as "MAP" if scale is small and is called "PLAN" if scale is large.
- In case of MAPS vertical distances can also be represented along with horizontal distances and direction with the help of Contours.

Note: DISTANCE: Linear measurement in horizontal plane is termed as distance

ELEVATION: Linear measurement in vertical plane is termed as Elevation

for eg.



DISTANCE ELEVATION

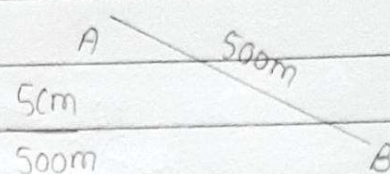
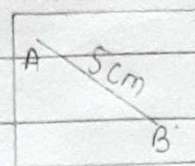
$$PR = 4 \quad PR' = 0$$

$$QR = 0 \quad QR' = 3$$

$$PQ = 4 \quad PQ = 3$$

SCALES

- Scale is the fixed ratio that every distance on the plan/map bears with the corresponding distance on the ground.



- Scale can be represented by any one of the following method

(A) ENGINEERING SCALE

- Here "1cm" on the plan represents some whole numbers of meters on the ground. Eg. 4cm = 500m

$$[1\text{cm} = 100\text{m}] \text{ ES}$$

$$1 = n \times 1 \text{ [RF]}$$

$$1:10,000$$

(B) REPRESENTATIVE FRACTION [RF]

- Here one unit of length on the plan represents some number of same units of length on the ground.
Eg. $1:50,000$ or $\frac{1}{50,000}$

Note :- Here distance is independent of units of measurement.

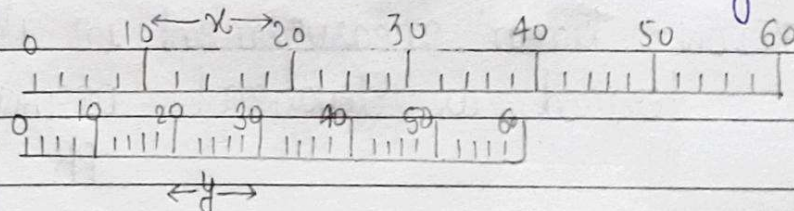
- ES, RF are both numerical scale

(C) GRAPHICAL SCALE

L-3

- If the plan or map is to be used after few years the numerical scale [ES, RF] may not give accurate result if the sheet or paper "shrinks" or "EXPAND" in such case graphical scale is used in which scale is drawn on the plan or map, which also shrinks or expand proportionally and the distances thus can be found accurately.

ORIGINAL SCALE
SHRINK SCALE

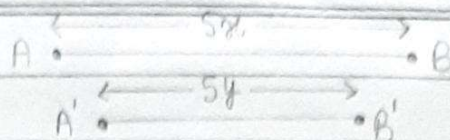


OS : $x \text{ mm} = 1 \text{ cm}$

AB on ground = 50 m

SC : $y \text{ mm} = 1 \text{ cm}$

A'B' on ground = 50 m



original scale : $\frac{x}{10,000}$

shrunk scale : $\frac{y}{10,000}$

$\frac{\text{shrunk length}}{\text{original length}} = \text{shrunk factor}$

Here the ratio of shrunk length to the original length is called as SHRUNK FACTOR / SHRUNK RATIO

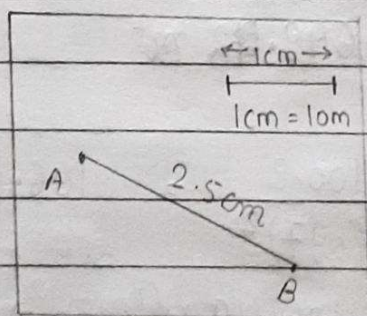
$$SF = \frac{SL}{OL} = \frac{y}{x} = \frac{\text{SHRUNK SCALE}}{\text{ORIGINAL SCALE}}$$

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Correct distance on map in terms of $\frac{\text{SHRUNK LENGTH}}{\text{SHRUNK FACTOR}}$

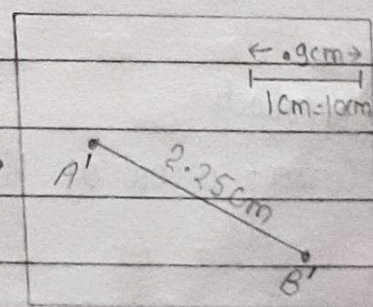
Correct area on map in terms of $\frac{\text{SHRUNK AREA}}{(\text{SHRUNK FACTOR})^2}$

Example :



ORIGINAL MAP

AFTER
2 YEARS



SHRUNK MAP

AB = 2.5 cm

A'B' = $2.5 \times 10 = 25 \text{ m}$

Shrunk scale = $OS \times \frac{y}{x}$

$$= .9 \times \frac{1}{10,000} = \frac{1}{10,000}$$

By Jaspal Sir

$$AB' = 2.25 \times 111.11 \\ = 2500 \text{ cm} = 25 \text{ m}$$

$$(ii) \text{ Correct distance} = \frac{SL}{SF} = \frac{2.25}{SF} \\ = \frac{2.25}{(.97)} = 2.50 \text{ m}$$

$$AB = 2.5 \times 10,000 = 2500 \text{ cm} = 25 \text{ m}$$

Q $\Rightarrow$ The area of the plan of an old survey plotted to a scale of 1 cm to 1 cm measure now 100.2 cm² as found by a planimeter. The plan is found to have shrunk so that a line originally 10 cm long now measure 9.7 cm only. Compute

(i) Shrink scale

(ii) True area of Survey. ESENOTES.COM

$$\text{Solution} \Rightarrow (i) \quad SF = \frac{y}{x} = \frac{9.7}{10} = .97$$

$$SC = OS \times SF = .97 \times \frac{1}{1000} = \frac{1}{1030.93}$$

1st approach

$$(ii) (a) \text{ True area of Survey} = 100.2 \times (1030.93)^2 \times 10^{-4} \\ = 10649.42 \text{ m}^2$$

2nd approach

$$(b) \text{ Correct area of the map} = \frac{100.2}{(.97)^2} = 106.49 \text{ cm}^2$$

$$\text{True area of Survey} = 106.49 \times (1000)^2 \times 10^{-4} \\ = 10649.42 \text{ m}^2$$

Question $\Rightarrow$ In 1950, plan of a rectangular field was drawn with a scale of $1\text{cm} = 40\text{m}$. The present dimension of the field read as $30\text{cm} \times 10\text{cm}$. If an original reference line of 9.4cm now reads 10cm then what is the actual area of field.

Solution $\Rightarrow$ Expanded factor = $\frac{y}{x} = \frac{10}{9.4} = 1.064$

$$ES = OS \times EF = \frac{1}{40 \times 10^2} \times 1.064 = \frac{1}{3760}$$

$$\text{Actual area of the field} = 30 \times 10 \times (3760)^2 \times 10^{-4} \\ = 424128 \text{ m}^2$$

Note: Error due to wrong measuring scale.

- If a wrong measuring scale is used to measure the scale of an already drawn line or area on the plan the measured length or area, will not be correct.

- Hence it is to be corrected as follows

$$\text{Correct length} = \frac{\text{RF of wrong scale}}{\text{RF of correct scale}} \times \text{measured length}$$

$$\text{Correct area} = \left[\frac{\text{RF of wrong scale}}{\text{RF of correct scale}} \right]^2 \times \text{measured area.}$$

Q. A Surveyor measured a distance b/wⁿ two points marked on the plan drawn to a scale of $1\text{cm} = 1\text{m}$ & found it to be 50m . Later he detected he used a wrong scale of $1\text{cm} = 50\text{m}$ for measurement. Determine the correct length

- What would be the correct area if the measured area is 50m^2 .

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- What would be the correct area if the measured area is 50 m^2 .

Solution (a) Correct length = $\frac{(1/50)}{(1/100)} \times 50 = 100\text{m}$

(b) Correct area = $\frac{(1/50)^2 \times 50}{(1/100)^2} = 200\text{m}^2$

CHOICE OF SCALE OF MAP

- The choice of scale of map to be selected depends upon

- (a) use of the map
- (b) extent of the territory to be represented

∴ while deciding the scale of map following points are considered.

(i) choose a scale large enough so that in plotting or in scaling distance from the finished map, it will not be necessary to read the scale closer than 0.25mm.

(ii) choose as small a scale as is consistent with a clear representation of smallest detail desired.

- Some common scale generally used in various survey are as follow.

PURPOSE OF SURVEY	SCALE
(a) TOPOGRAPHIC SURVEY	1cm = 10m - 2.5 Km
(b) CADASTRAL SURVEY	1cm = 5m - 0.5 Km
(c) GEOGRAPHICAL SURVEY	1cm = 5 Km - 160 Km

Types of SCALE

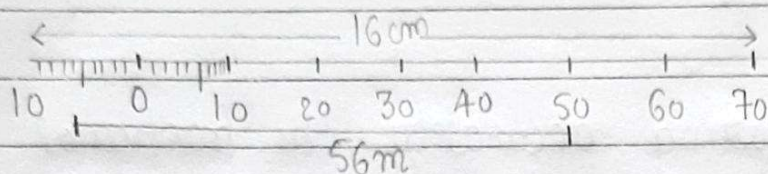
- Scales may be classified as follow

(A) PLANE SCALE

- It is a scale on which it is possible to measure two dimension only
eg- cm & mm, m & dm [unit & tenth]

Ex: 1cm = 5mm . plane scale

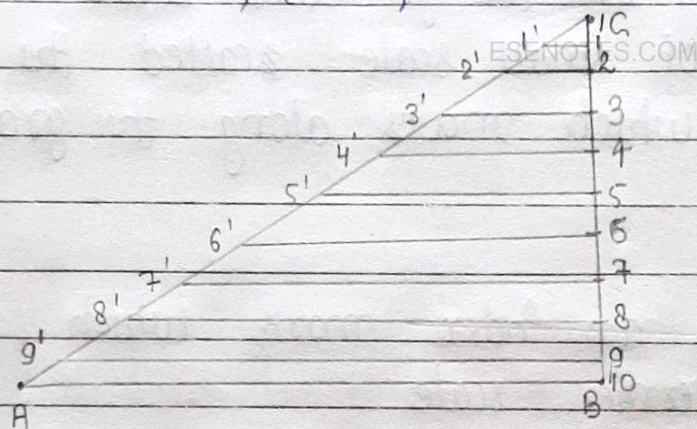
$$56m = 50 + 6dm$$



(B) DIAGONAL SCALE

L-4

- on this scale it is possible to measure three dimension
i.e unit, tenth, hundred . eg. m, dm, cm



In $\Delta C'11$ and ΔCAB

$$\frac{C1}{CB} = \frac{11}{AB}$$

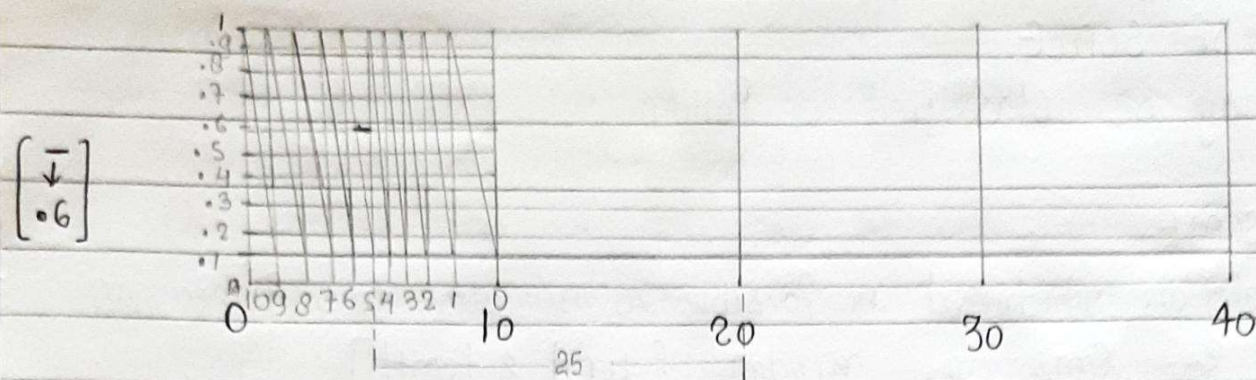
$$\frac{1}{10} = \frac{11}{AB}$$

$$\boxed{11' = \frac{AB}{10}}$$

Similarly

$$\frac{22'}{10} = \frac{2AB}{10}$$

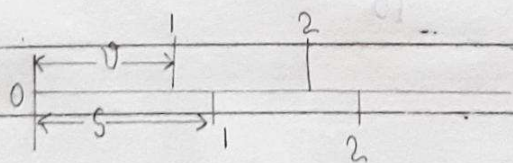
$$\frac{99'}{10} = \frac{9AB}{10}$$



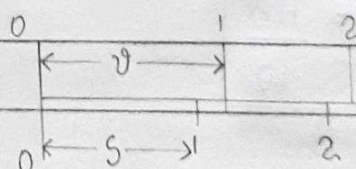
Eg - 25.6

(iii) VERNIER SCALE

- It is a device measuring accurately the fractional part of ~~dev~~ the smallest division on graduated scale or main scale.
- Hence in this case Readings are taken closer than the smallest reading on the graduated scale
- The Vernier Consist of small scale called the Vernier scale which moves along the graduated scale/main scale
- The Vernier scale has an Index mark which represent zero of Vernier scale
- The divisions of Vernier scale are made either made slightly shorter or longer than that of main scale.
- The Least Count of the Vernier is Equal to difference of length of one division of the main scale and one division of Vernier scale



$$LC = S - V$$



$$LC = V - S$$

Date

- Vernier scale are of following type

(A) DIRECT VERNIER

- In this case divisions on Vernier scale are slightly shorter than those on main scale
- In this case readings on both main and Vernier scale increases in same direction.

- Let n divisions on the Vernier scale are equal to in length of $(n-1)$ divisions of main scale

$$nV = (n-1)S$$

$$V = \frac{(n-1)S}{n}$$

V = length of one division on Vernier

S = ee ee ee ee ee main scale.

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$$LC = S - V$$

$$= S - \frac{(n-1)S}{n} = \frac{nS - nS + S}{n}$$

$$\boxed{LC = \frac{S}{n}}$$

(B) DOUBLE VERNIER

- It is the vernier in which main scale is figured in both the direction and Vernier also extends on both side of index mark

- Here also $LC = \frac{S}{n}$

(C) RETROGRADE VERNIER

- In this type of Vernier divisions are slightly longer than those of main scale
- Here Readings in the case of Retrograde Vernier increase in a direction opposite to that of main scale
- Let n divisions of Vernier scale are equal to $(n+1)$ divisions of main scale

$$nV = (n+1)S$$

$$V = \frac{(n+1)S}{n}$$

$$LC = V - S$$

$$= \frac{(n+1)S}{n} - S = \frac{nS + S - Sn}{n}$$

$$LC = \frac{S}{n}$$

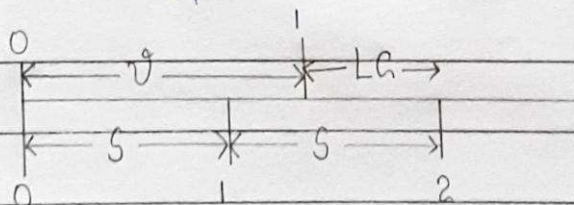
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(D) EXTENDED VERNIER

- It may happen that the divisions on the main scale are very close and it would be difficult, if the vernier are of normal length to judge the exact graduation where coincide occur

In this case Extended Vernier is use

- Here $2n-1$ divisions on main scale are equal to n divisions of Verniers



$$(2n-1)S = nV$$

$$V = \frac{(2n-1)S}{n}$$

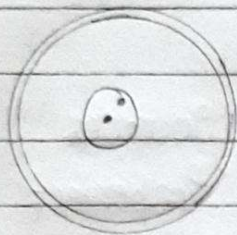
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$$LC = \sum S - \bar{S} = \sum S - \frac{(\sum n - 1)S}{n} = \frac{n \sum S - \sum n S + S}{n}$$

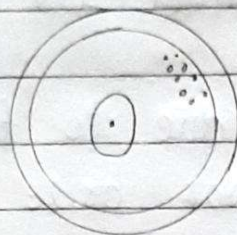
$$LC = \frac{LC}{n} = \frac{S}{n}$$

Lec-05

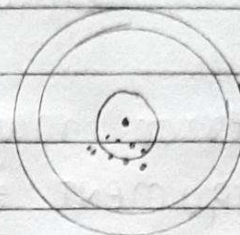
ACCURACY AND ERROR



ACCURATE VALUE



PRECISE VALUE



ACCURATE & PRECISE VALUE

(i) PRECISION

Degree of Perfection or degree of fineness used in the instrument, methods of observation and observation is termed as Precision.

(ii) ACCURACY

DEGREE of Perfection obtained in using Instruments, method of observation and observation is termed as accuracy.

Note: The difference between two measured value of same quantity is termed as "DISCREPANCY"

- It is not same as an error
- A small discrepancy may have greater error and a large discrepancy may have small error.

(iii) ERROR

It is the difference between a measured value and true value of quantity

- Since true value of the quantity is not known, true error of measurement cannot be found, hence measurement is done in a certain limits of error prescribed.

$$\text{ERROR} = \text{MEASURED VALUE} - \text{TRUE VALUE}$$

$$\text{CORRECTION} = \text{TRUE VALUE} - \text{MEASURED VALUE}$$

Hence $\text{ERROR} = (-) \text{CORRECTION}$

- These error may arise due to following sources

(A) INSTRUMENT: This error arise due to imperfect or faulty instrument used for the measurement.

Eg: A chain/tape being used that is too short or too long than actual.

(B) PERSONAL ERROR: This error arise due to imperfection in human response like weak eye sight, imperfection of touch etc.

(C) NATURAL ERROR: These error arise due to natural phenomenon such as temperature, humidity, wind, gravity, refraction, magnetic declination

- Error are classified into following.

(A) MISTAKE

- These are error which arise from inattention, inexperience, carelessness, poor judgement, confusion in mind of observer.

- It can be identified by taking same measurement by another observer.

- It can also be overcome by comparing several measurement of same quantity & discard the odd value.

- Eg. A reading of 7m is booked as 17m
- Miscounting of the number of tape length

(B) SYSTEMATIC [CUMMULATIVE ERROR] ERROR

- It arises from the source that act in a similar manner on observation.
- The method of measurement, the instrument used, physical condition at the time of measurement cause systematic error.

Eg. Expansion of steel tapes
using Compass in a particular magnetic field.

- The errors are Cumulative in nature.
- These can be eliminated by applying corrections.
- The proper way to check systemic error is to measure the same quantity by an entirely different method.

(C) ACCIDENTAL / RANDOM / COMPENSATORY ERROR

These are the errors remaining after mistakes and systemic errors are removed

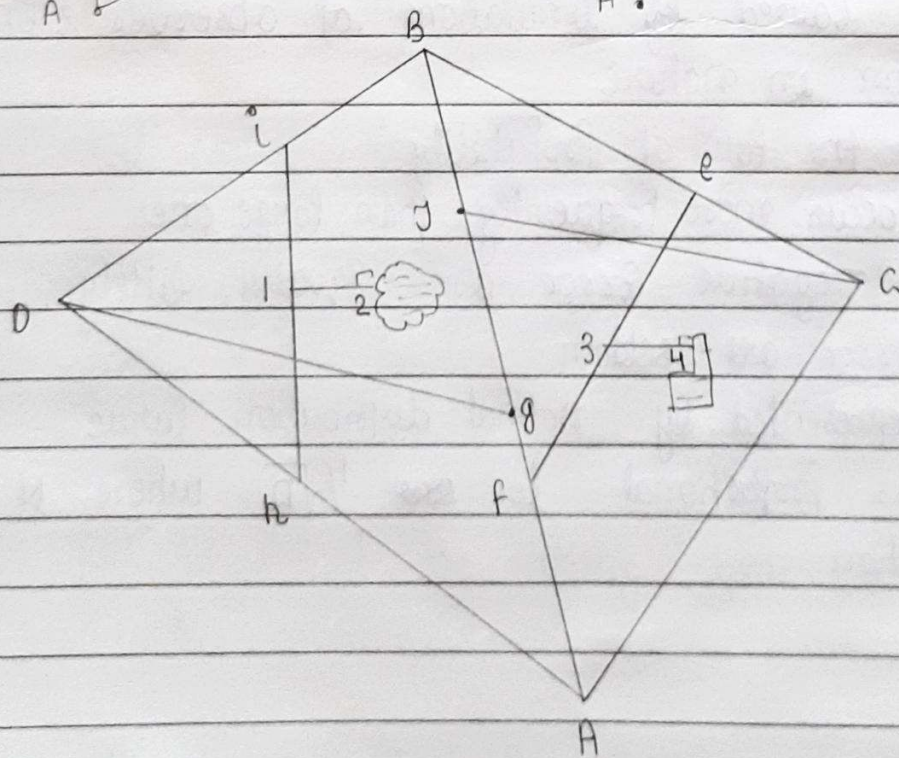
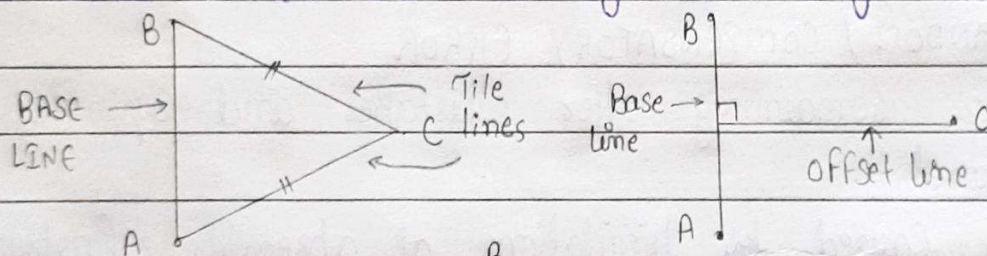
- It is mainly caused by limitations of observer & instrument and are random in nature.
- These errors follow the law of Probability
 - (i) Small error occur more frequently than large ones
 - (ii) Positive and negative error are equally likely
 - (iii) Very large error are seldom
 - (iv) It can be represented by normal distribution curve.
 - (v) Random error is proportional to $\frac{1}{\sqrt{N}}$ where N is no of observation

LINEAR MEASUREMENT

- Linear measurement can be obtained by chain surveying in which chain or tape is used
- It is suitable for small ^{open} areas having few simple details
- These details may be classified as
 - (A) HARD DETAIL: Buildings, Roads, Walls
 - (B) SOFT DETAIL: RIVER, TREES
 - (C) OVERHEAD DETAIL: POWER AND TELEPHONE LINES
 - (D) UNDERGROUND DETAIL: WATER AND SEWER PIPES

Note: chain surveying consists of measuring a length of series of straight lines with chain and then locating details on the ground relative to ~~the~~ these lines.

- The details are located by measuring two other lines termed as tie lines or by measuring offset



(i) MAIN STATION: It is a point of chain survey where the two sides of triangle met.

- These station act like Control Point or boundary of Survey area, A, B, C, D

(ii) TIE / SUBSIDIARY STATION: ~~Station~~ It is the point on the line joining two main station

- It is used for locating interior details of the area.
e, f, g, h, i, j

(iii) MAIN SURVEY LINE: line joining the two main survey station is termed as main Survey line

AB, BC, CD, DA, AC, AD

(iv) TIE / SUBSIDIARY / AUXILIARY LINE: It is the line joining two tie station

It is used to locate interior detail of survey area, which is far away from main ^{survey} line
e, f, g, h

(v) BASE LINE: It is the longest main Survey line which passes through the centre of the survey area.

- It is the important Survey line as the location of rest of survey line is fixed on the basis of this line only.
- Eg - AB

(vi) CHECK LINE / PROOF LINE: It is provided to check the accuracy of field work

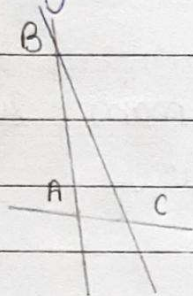
- The measured length of check line and the computed line from the plan must be same

Eg - C, D

(vii) OFFSET: It is the distance of object from survey line
 - It may be perpendicular or oblique
 Eg. 12, 34

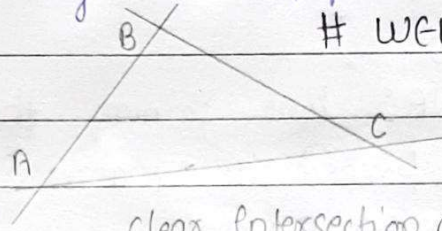
(viii) CHAINAGE: It is the distance measured along the main survey line in the direction of progress of work.
 - It is generally used in linear construction projects.
 Eg. road, railways, water pipe line, tunnel

WELL
CONDITION
TRIANGLE



No clear intersection
 at main station B

WELL CONDITION TRIANGLE



clear intersection at main station B

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- In chain surveying efforts should be made to select main survey stations such that they form "WELL CONDITION TRIANGLE"
- It is a triangle if all intersection of lines are clear for plotting
- If the angle b/w the lines is $30^\circ - 120^\circ$ clear intersection is achieved.
- An equilateral triangle is most appropriate well condition triangle.

EQUIPMENT USE IN CHAIN SURVEYING

(i) SURVEYING CHAIN

used for measuring dist. where high accuracy is not req.

- It consist of large links made of galvanized mild steel wire of 4mm diameter

- To enable the reading of fraction of chain length with difficulty brass tallies / tags are provided at fixed interval [for 10m surveying chain is provided after 1m interval]

Types of chains available are as follows:

- REVENUE CHAIN : 33 ft (16 links)
- GUNTER'S CHAIN : 66 ft (100 links)
- METRIC CHAIN : ~~for~~ 30m (150 links), 20m (100 links)
- ENGINEERING CHAIN : 100 ft (100 links)

Note: 1 mile = 80 Gunter's chain

1 acre = (10 Gunter's chain)²

Note: As per IS specification every metre of length should be individually accurate to within $\pm 2\text{mm}$ when measured under a tension of 80N

- The overall length of chain should be within following limits

- $20\text{m} \pm 5\text{mm}$ for 20m chain
- $30\text{m} \pm 8\text{mm}$ for 30m chain

(B) TAPE

- These tapes are easy to handle and more precise than chains
- depending upon the material used these tapes are of following type

(i) CLOTH / LINEN TAPE

Made up of synthetic material

- length 10-30m, width 12-15mm
- It get shrink in the presence of moisture
- It gets damaged when stretched more
- It is lightly to twist during measurement

(ii) METALLIC TAPE

- It is linen tape reinforced with brass or copper wire
- Its accuracy is more than linen

(iii) STEEL TAPE

- It is more accurate than metallic tape
- length 50m & width 10mm
- It is affected by temp & moisture

(iv) INVAR TAPE

- It is made up of alloy of Nickel & steel
- It is highly precise
- It is less affected by temp [$\alpha = 1.22 \times 10^{-6}/^{\circ}\text{C}$]
- It is soft and deforms easily

(C) PEGS

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- These are used to mark the definite point on the ground from which measurement are to be taken or any instrument is to be set

(D) ARROW

- While measuring the length of long line chain or tape is to be laid no of times and the position of end is marked with the help of arrows

(E) RANGING ROD

- The process of locating no of points on a long survey line is termed as Ranging
- These rods are used to locate intermediate points such that these points lie on straight line joining the end stations

Date.....

Length: 2-3m, diameter: 30mm, Painted as alternate Red/ Black and white bands of 200mm length

(Vi) OFFSET ROD

These are similar to ranging rod with only difference, that they are provided with hook or opening at the top and two narrow vertical slots at right angle to each other at eye level.

- It is used to align the offset line

(Vii) CROSS STAFF

It is a simple instrument used for setting out of offsets to the chain line from a given point.

- It is also used for setting out right angle at a given point on survey line
- In order to set out angles in multiple of 45° "French Cross Staff" can be used. accuracy of which is less than cross staff
- Note: In order to set out any desired angle from survey line "adjustable cross ~~line~~ staff" can be used.
- It consists of 2 cylinders of equal diameter which can rotate over each other
- Lower cylinder consists of main & sub-main scale & upper cylinder consists of vernier scale and a opening (slit) to provide a line of sight

(Viii) OPTICAL SQUARE

- It is a small instrument used to set out right angles and is more convenient and accurate than cross staff

- It consists of two mirrors termed as horizon (H) and Index (I) placed at an angle of 45°
- The mirror (H) is ~~partly~~ silvered on the top and unsilvered at the bottom where mirror "I" is fully silvered
- It works on the principle of "double Reflection" i.e. if a ray of light is reflected or undergoes reflection successively then the angle b/wⁿ the incident & reflected light is twice the angle b/wⁿ the reflecting surface.

Note: Instead of using two mirrors, a prism can also be used having two reflecting sides at an angle of 45° .
And the unit is termed as "PRISM SQUARE"

(ix) SIMPLE CLINOMETER

It is used for measuring the angle of slope of a ground.

- A light Plumb bob is suspended from the centre of the semi-circle
- A flat base is attached to the semi-circle to lay the clinometer on the surface whose slope is to be measured

(x) LINE RANGERS

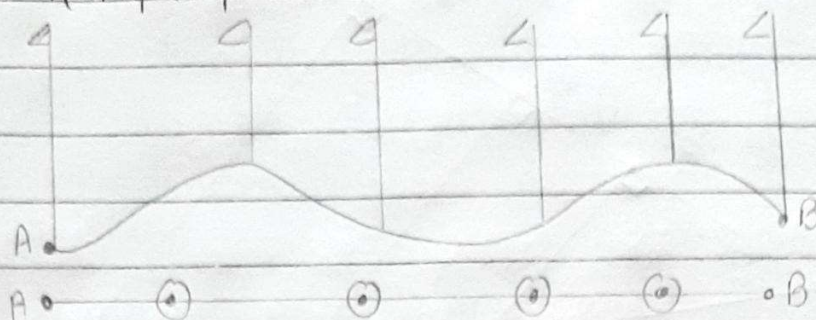
- It is a device which is used to set intermediate point on survey lines
- It consists of two right-angled ^{isosceles} triangular prisms placed one over each other
- The diagonal of these two prisms are painted silvered to form the reflecting surface
- The process of ranging can be done by any of the following methods

(a) DIRECT RANGING

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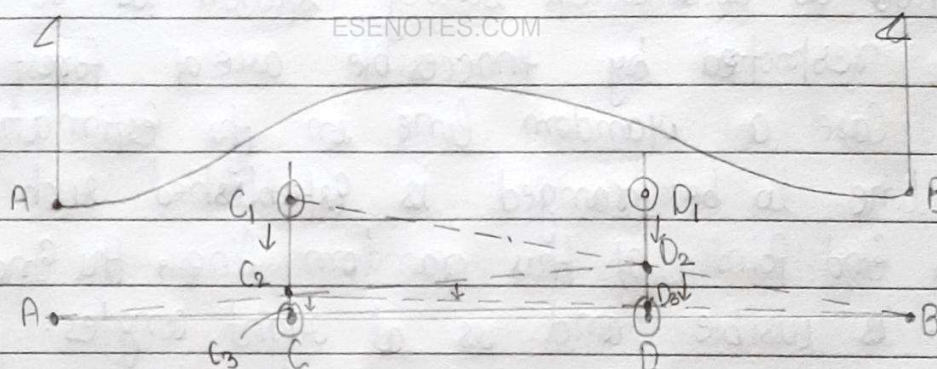
Ranging can be carried out by following mtd.

(A) DIRECT RANGING



- When ranging rods are placed on intermediate points along the chain line by direct observation from either of the end station, is termed as "DIRECT RANGING"

(B) INDIRECT RANGING

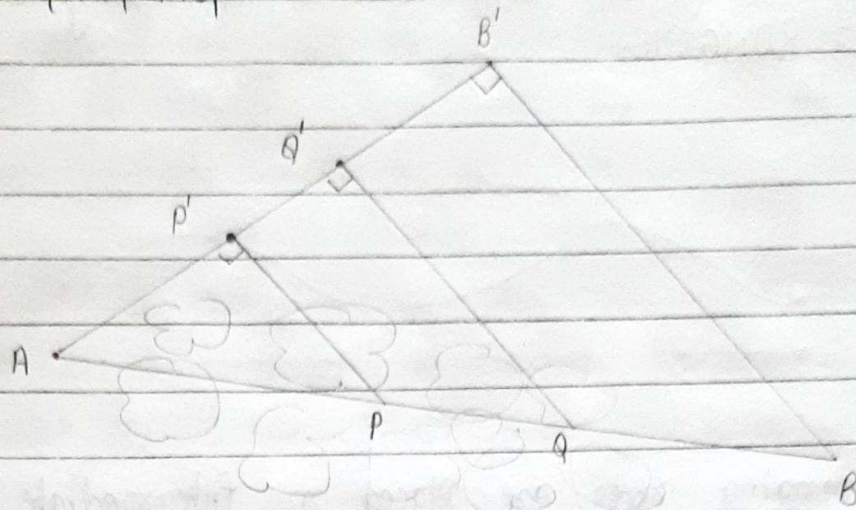


When the end stations are not visible due to rising ground level between them or due to long distances between end points,

Indirect ranging is adopted as follows

- In this case 2 chainman stands at some arbitrary point C, & D, in b/w the two end station A, B such that chainman at C, can see both D, & B and chainman at D, can see both C, & A
- Now these both chainman direct each other towards the survey line AB upto an extent they reach the intermediate point CD on survey line AB

(C) RANDOM LINE RANGING



$$AB = \sqrt{AP'^2 + B'B'^2}$$

$$\frac{AB'}{AP'} = \frac{BB'}{PP'}, \quad \frac{AB'}{AQ'} = \frac{BB'}{Q'Q'}$$

- This Method is used when survey station to be ranged are restricted by inaccessible area of forest.
- In this case a random line in the estimation direction of line to be ranged is established such that from the end point of this random line, the end survey station is visible and is at right angles.
- Then ranging of intermediate points done using similar triangles.

CHAINING

- Linear measurement with means of chain is termed as chaining
- It can be done by any of the following mtd.

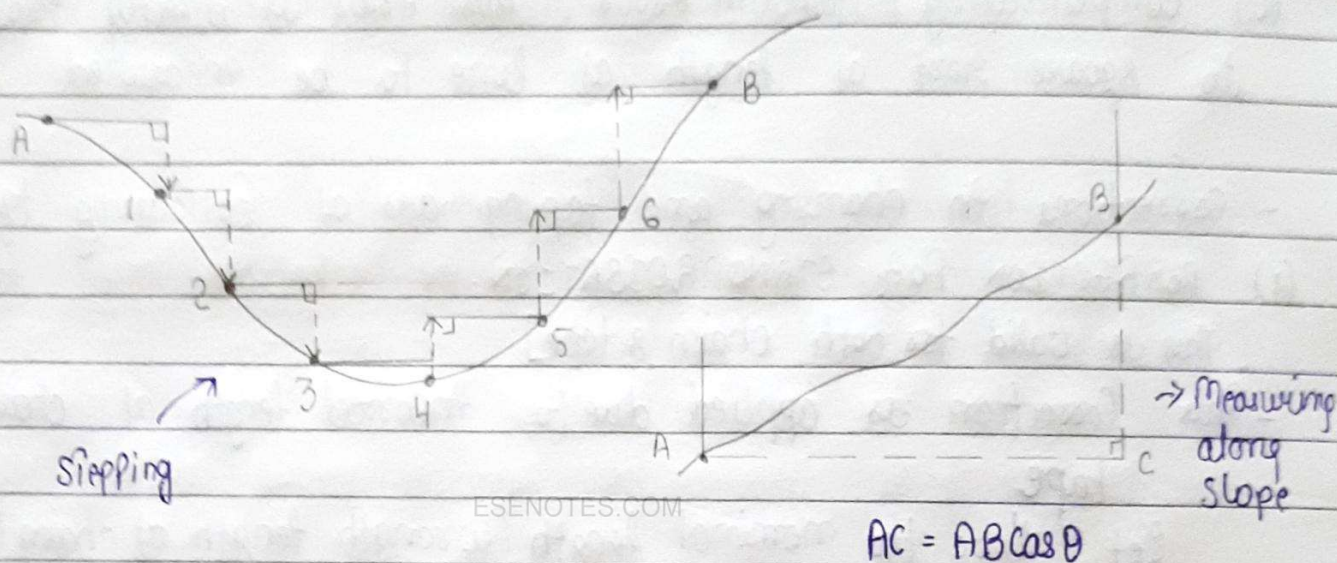
(a) ON SMOOTH LEVEL GROUND

- For measuring a distance longer than one chain length it is necessary to establish intermediate point via ranging
- For this purpose two persons are required i.e. a leader & a follower
- Leader moves forward along the survey line & goes on inserting

Axes at the end of chain and follower keeps collecting it.

- Thus at the end of measurement number of axes collected by the follower indicates the number of chain length measured.

(B) ON SLOPING GROUND



- In plane surveying the distance measured should be horizontal
- But in certain cases the change in elevation from the end of chain to another end is large enough that chain cannot be kept horizontal.
- In such cases two methods were used:
 - (A) STEPPING: Use when slope is variable
 - (B) MEASURING ALONG SLOPE: Use when slope is constant.

ERRORS AND CORRECTION IN CHAINING / TAPING

- Error is defined as difference of measured length & true length

$$\text{Error} = \text{Measured length} - \text{True length}$$

$$\text{True length} = \text{Measured length} - \text{Error}$$

$$\text{Correction} = \text{True length} - \text{Measured length}$$

$$\text{True length} = \text{Measured length} + \text{Correction}$$

- Errors in chaining can be of following type

(a) MISTAKE

(b) CUMULATIVE / SYSTEMATIC ERROR: This error is directly proportional to length of the line and may be positive or negative.

(c) Compensating / Random error: This error is directly proportional to square root of length of line to be measured.

- Corrections in chaining and taping are of following types.

(i) CORRECTION FOR STANDARDISATION.

[This is valid for both chain & tape]

- This correction is applied due to incorrect length of chain or tape

Let L' be the nominal length / wrong length of chain / tape

L be the true length of chain

L' wrong length of line measured

L be correct length

$$\text{Correct length } (L) = \frac{L'}{L} \times L$$

Correct length = True length of chain (L) $\times$ No. of times the chain was used for measurement of length of line

$$L = L \times \frac{L'}{L}$$

$$L = \left(\frac{L}{L'} \right) \times L'$$

Note: The same concept can be used for area & volume

$$\text{True area} = \left(\frac{L}{L'} \right)^2 \times \text{measured area}$$

$$\text{True volume} = \left(\frac{L}{L'} \right)^3 \times \text{measured volume}$$

Correction for standardisation

$C_s = \text{True length} - \text{Measured length}$

$$C_s = L - L'$$

$$C_s = \left(\frac{L}{L'} \right) L' - L'$$

$$C_s = L' \left[\frac{L}{L'} - 1 \right]$$

$$C_s = \left[\frac{L - L'}{L'} \right] L'$$

Question: A line of true length 400m when measured by a 20m tape is reported to be 401m. The correct length of tape is

Solution) $L = 400$ $L' = 20\text{m}$ $L' = 401$

$$L = L' \times \frac{L}{L'}$$

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$$400 = L' \times \frac{401}{20}$$

$$L' = 19.95\text{m}$$

Q) A plan of an old survey plotted to a scale of 1cm to 10m carried a note stating that, the chain (20m) was used was .8 link (16) to short. It was also found that the plan had shrunk so that a line originally 10cm long was 9.77cm. The area of the plot on the available plan was found to be 58.2 cm². What is the correct area in the survey in hect.

A) $L' = 20\text{m}$ $L = 20 - .16 = 19.84\text{m}$

$$RF = \frac{1}{1000}\text{cm}$$

$$SF = \frac{L}{L'} = \frac{9.77}{10} = .977$$

$$\text{Actual area before shrink} = \frac{58.2}{SF^2} = \frac{58.2}{(0.977)^2} = 60.97 \text{ cm}^2$$

$$\begin{aligned} \text{Area of survey} &= 60.97 \times (10^3)^2 = 6.097 \times 10^7 \text{ cm}^2 \\ &= 6.097 \times 10^7 \times 10^{-4} \times 10^{-4} \\ &= 0.6097 \text{ hect.} \end{aligned}$$

$$\begin{aligned} \text{Correct area of survey} &= \left(\frac{l}{l'}\right)^2 \times A \\ &= \left(\frac{19.84}{20}\right)^2 \times 0.6097 \\ &= 0.59998 \text{ hect} \end{aligned}$$

Q) A 200m chain was found to be 10cm too long after chaining a distance of 200m. It was found to be 18cm too long at the end of the day's work after chaining a total distance of 400m. What is the true distance if the chain was correct before the commencement of the work.

Solution) For 1st 200m

$$l' = 20\text{m}$$

$$l = \frac{20 + 20.1}{2}$$

$$l = 20.05\text{m}$$

$$L = \frac{l}{l'} \times L'$$

$$L = \frac{2000}{20} \times 20.05$$

$$L = 2005\text{m}$$

For 2nd 200m

$$l' = 20\text{m}$$

$$l = \frac{20.1 + 20.18}{2}$$

$$l = 20.14\text{m}$$

$$L = \frac{2000}{20} \times 20.14$$

$$L = 2014\text{m}$$

$$L = L_1 + L_2 = 2005 + 2014 = 4019\text{m}$$

(ii) CORRECTION FOR SLOPE (C_g)

[It is valid for both chain and tape]

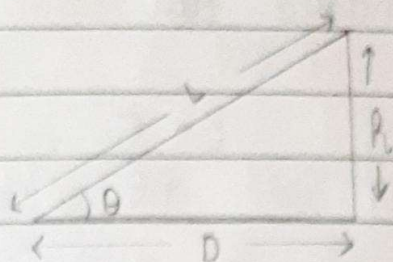
$$C_g = D - L$$

(a) when slope angle (θ) is known

$$C_g = L \cos \theta - L$$

$$C_g = -L(1 - \cos \theta)$$

Note: slope correction will always be negative



(b) when difference of Elevation is known (h)

$$C_g = D - L$$

$$C_g = \sqrt{L^2 - h^2} - L$$

$$C_g = L \left[\left(1 - \frac{h^2}{L^2} \right)^{1/2} - 1 \right]$$

$$C_g = L \left[1 - \frac{1}{2} \frac{h^2}{L^2} - \frac{h^4}{8L^4} + \text{other higher terms} \right] - L$$

Neglecting higher terms of h

$$C_g = L \left[1 - \frac{h^2}{2L^2} - 1 \right]$$

$$C_g = -\frac{h^2}{2L}$$

Note: If higher power of h is not neglected then this correction increases by $\frac{h^4}{8L^3}$

- Now all the further correction are applicable for "TAPE" only

(iii) CORRECTION FOR PULL/TENSION

- If the pull applied to the tape during measurement is more than the standard pull at which tape was standardised then the length of tape increases hence the distance measured become less than Actual

- Thereby the Correction applied is Positive
- This Correction is negative if the applied Pull is less than standard Pull

$$E = \frac{\sigma}{\epsilon} = \frac{A\sigma}{\epsilon} = \frac{P - P_s}{A C_p / L}$$

$$C_p = \frac{(P - P_s)L}{A \cdot E}$$

P_s = standard Pull

P = Applied Pull

A = cross-section area of tape

E = Modulus of elasticity of tape

L = Measured length

(10) CORRECTION FOR TEMPERATURE (C_t)

- The length of tape changes due to change in temp while taking the measurement
- Correction for temp. is (+)ve, if the temperature during measurement is more than that of standard temp. and (-)ve if the temp during measurement is less than that of standard temp.

$$\Delta L = \alpha (\bar{T}_m - T_0) L$$

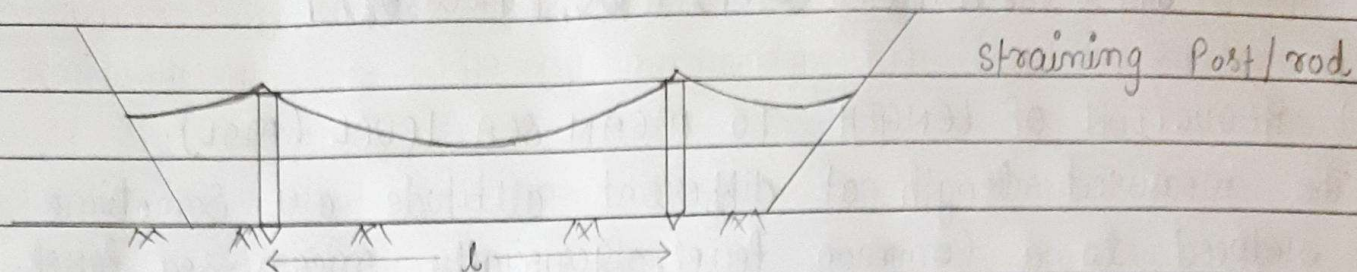
$$C_t = \alpha (\bar{T}_m - T_0) L$$

α = Coeff. of thermal Expansion

$\bar{T}_m$ = Mean temp during measurement

T_0 = Temp. during standardisation

(11) CORRECTION FOR SAG (C_{sg})



- If higher accuracy is required in taping, in that case tape must be hung in catenary and correction for the sag in tape is applied

Case (a) : If the tape has been standardized on flat & used in catenary, sag correction is (-)ve

Case (b) : If the tape has been standardized on catenary & used in flat, sag correction is (+)ve.

- For sag correction the profile is considered to be parabolic

$$C_{sg} = \frac{w^2 l^3}{24 p^2}$$

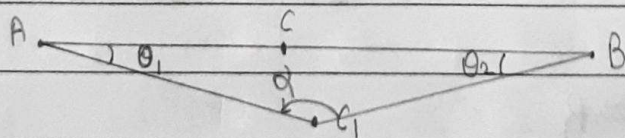
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w = weight of tape per unit length

l = length of tape suspended b/w support

p = applied pull

(vi) CORRECTION FOR MISALIGNMENT (C_m)

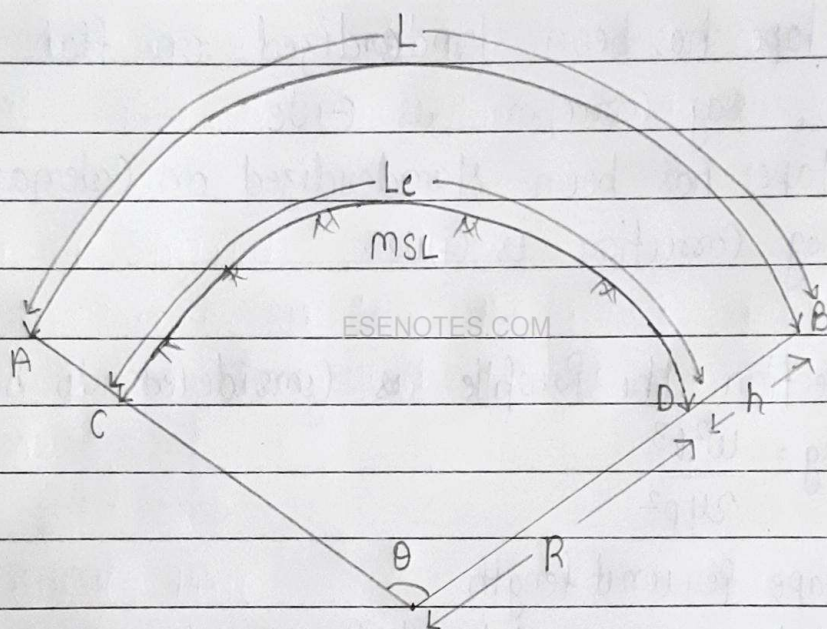


- If the survey line is not accurately ranged out, the error due to misalignment occurs
- In this case the measured distance will always be greater than the actual, hence error is positive and correction is negative

$$C_m = -[AC_1(1 - \cos \theta_1) + BC_1(1 - \cos \theta_2)]$$

(U11) REDUCTION OF LENGTH TO MEAN SEA LEVEL (MSL)

- The measured length at different altitude are sometime reduced to a common level, generally mean sea level.
- As the Earth is oblate spheroidal in shape the measured distance at any altitude h above the MSL would be greater than the equivalent length at mean sea level.



$$\theta = \frac{L}{R+h} = \frac{L_c}{R}$$

$$L_c = L \left[\frac{R}{R+h} \right]$$

$$C_h = \frac{L R}{R+h} - L$$

$$C_h = - \frac{hL}{R+h}$$

Since $R \approx 6400 \text{ km}$

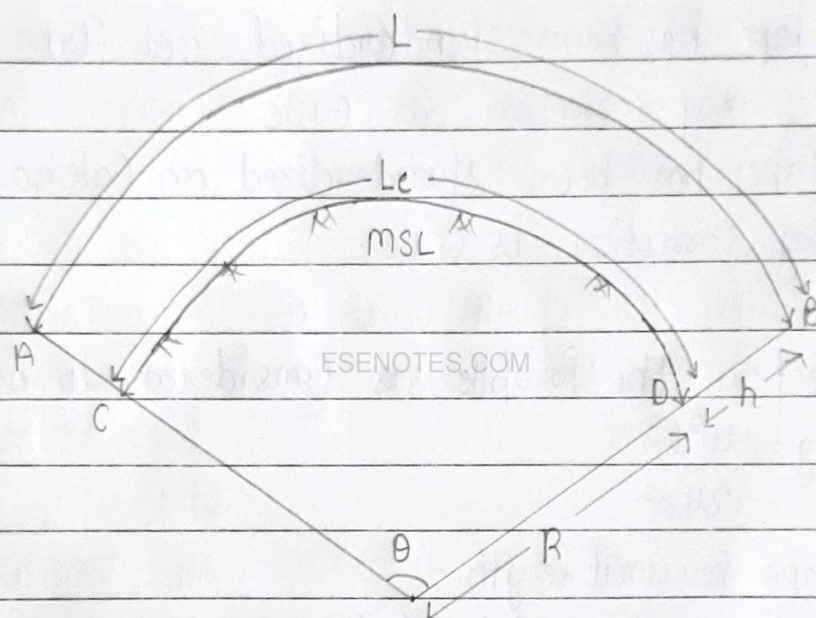
$$\text{i.e. } R \gg h \Rightarrow R+h \approx R$$

$$C_h = - \frac{hL}{R}$$

$$C_m = -[AC_1(1 - \cos \theta_1) + BC_1(1 - \cos \theta_2)]$$

(VII) REDUCTION OF LENGTH TO MEAN SEA LEVEL (MSL)

- The measured length at different altitude are sometime reduced to a common level, generally mean sea level.
- As the Earth is oblate spheroidal in shape the measured distance at any altitude h above the MSL would be greater than the equivalent length at mean sea level.



$$\theta = \frac{L}{R+h} = \frac{L_e}{R}$$

$$L_e = L \left[\frac{R}{R+h} \right]$$

$$C_h = \frac{LR}{R+h} - L$$

$$C_h = -\frac{hL}{R+h}$$

Since $R \approx 6400 \text{ km}$

i.e. $R \gg h \Rightarrow R+h \approx R$

$$C_h = -\frac{hL}{R}$$

Question) A steel tape of Nominal length 30m was used to measure a line PQ by suspending it between supports. If the measured length was 29.861m when the slope angle was $3^\circ 45'$ and the mean temp. and tension applied were respectively 10°C & 100N, determine the Corrected horizontal length.

The standard length of tape was 30.004m at 20°C and 44.5 N tension. The tape wt. is $.16\text{N/m}$ & X-X area is 2mm^2 , $E_c = 2 \times 10^5 \text{N/mm}^2$, $\alpha = 1.12 \times 10^{-5}/^\circ\text{C}$.

Ans) i) $L = 30\text{m}$ $L' = 29.861\text{m}$ $\theta = 3^\circ 45'$ $L_s = 30.004$

$$C_s = L' \left(\frac{L - L'}{L} \right) = 29.861 \left[\frac{30.004 - 30}{30} \right]$$

$$C_s = .00398\text{m}$$

(ii) $C_g = -L(1 - \cos\theta)$

$$= -29.861(1 - \cos 3^\circ 45') = -0.064\text{m}$$

(iii) $C_p = \frac{(P - P_0)L}{AE} = \frac{(100 - 44.5) 29.861}{2 \times 10^{-6} \times 2 \times 10^5 \times 10^{-6}} = .004\text{m}$

(iv) $C_t = L\alpha(T_m - T_0) = 1.12 \times 10^{-5} \times 29.861(10 - 20) = -.003\text{m}$

(v) $C_{sg} = \frac{-w^2 L^3}{24P^2} = \frac{.16^2 \times (29.861)^3}{24(100)^2} = -.00284\text{m}$

$$\text{Total Correction} = .00398 - .064 + .004 - .003 - .00284 = -0.0619$$

$$\text{Correct horizontal distance} = 29.861 - .0619 = 29.798\text{m}$$

Date.....

OPERATION [FIELDWORK] IN CHAIN SURVEYING

- The operation involved in chain surveying includes

(a) RECONNAISSANCE

- It is the preliminary inspection of the area to be surveyed to get some idea of the terrain and identifying the principle features of the ground. (Moundain, rivers, valleys)

- In this ground is examined & decided upon for best possible arrangement of the triangle and best possible Marking of main survey station and survey line

(b) MARKING OF STATION

- once Reconnaissance is done, the main station are being marked on ground such that stations can be readily located afterwards

(c) RUNNING SURVEY LINE

- Survey lines are run to measure the distance between main stations and locate the adjacent details by offset

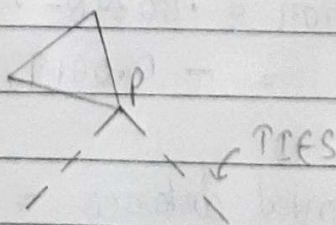
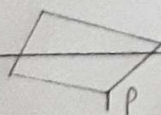
(d) TAKING OFFSET

- It is defined as lateral distance measured from survey line to the station whose detail is to be plotted

- These offset are of two types

(i) PERPENDICULAR OFFSETS / OFFSET

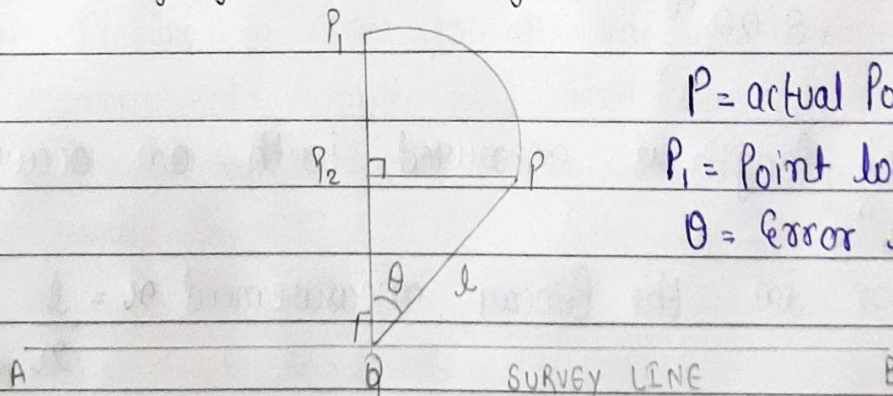
(ii) OBLIQUE OFFSET / TIES



Note: MAX / LIMITING LENGTH OF OFFSETS

- The Max length of offset is generally determined for the consideration that the error produced in plotting the detail should not exceed $.25\text{mm} / .025\text{cm}$
- $.25\text{mm}$ is the minimum value that the human eye can differentiate. hence a good draughtsman can distinguish a distance of $.25\text{mm}$ on paper
- Hence Maximum length of offset depends upon
 - (a) Accuracy desired
 - (b) scale of plotting
 - (c) Possible errors in direction & length
- Errors in laying offset may be due to
 - (a) Error only in laying offset
 - (b) Error in laying & linear measurement.

(a) Error in Laying offset only



P = actual Point on ground
 P_1 = Point located on paper
 θ = Error in laying direction

Length of Error on Ground = $PP_2 = l \sin \theta$

if scale of map is $1\text{cm} = S\text{m}$

Length of error on Paper = $\frac{l \sin \theta}{S} \text{ cm}$

For limiting length of offset, length of error on Paper should not less than $.025\text{cm}$ $\neq .025\text{cm}$ $\neq .025\text{cm}$

$$\frac{l \sin \theta}{S} < .025 \Rightarrow l < \frac{.025 \cdot S}{\sin \theta}$$

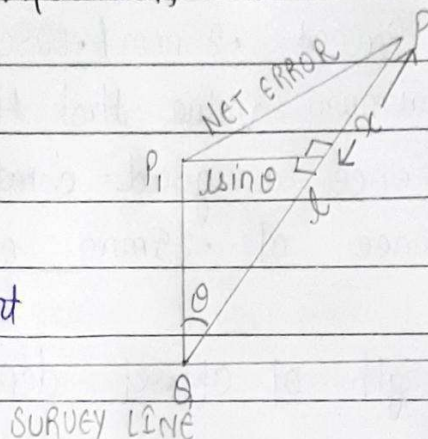
(B) ERROR IN LAYING DIRECTION AND LINEAR MEASUREMENT BOTH

P = actual Pt on ground

P_i = P_i located on plan

θ = Error in laying direction

x = Error in linear measurement



Length of error on ground will be $PP_i = \sqrt{(l \sin \theta)^2 + x^2}$

if scale of map is 1 cm = 5 cm

Length of error on plan = $\sqrt{(l \sin \theta)^2 + x^2}$

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For limiting length of offset : Length of error on plan < 0.025 cm

$$\frac{\sqrt{(l \sin \theta)^2 + x^2}}{S} < 0.025 \text{ mm}$$

$$l < \frac{1}{\sin \theta} \sqrt{(0.025 \cdot S)^2 - x^2}$$

Note : If offset length is measured with an accuracy of "1 in 1000"

Then error in the linear measurement $x = \frac{l}{1000}$

Q → If scale of drawing is

(i) 1 cm = 60 m

(ii) 1 cm = 5 m

for above cases, find out the limiting length of offset

If max error allowed in laying direction of offset

is 6°

$$A) \quad l = \frac{.025 \cdot S}{\sin \theta}$$

$$(i) \quad S = 60 \Rightarrow l = 14.35m$$

$$(ii) \quad S = 5 \Rightarrow l = ~~1.19m~~ 1.19m$$

Q) If max length of an offset allowed is 20m and max error allowed in length measurement is 25cm, scale of drawing is 1cm = 50m. Find out max error that can be allowed in laying direction of offset.

$$A) \quad l = 20m \quad ~~S = 50m~~ \quad S = 50m$$

$$l = \frac{1}{\sin \theta} \sqrt{(.025S)^2 - x^2}$$

$$20 = \frac{1}{\sin \theta} \sqrt{(.025 \times 50)^2 - (.25)^2}$$

$$\theta = 3.51^\circ$$

Q → An offset is measured with an accuracy of 1 in 100. If the scale of plotting is 1cm = 10m & the displacement of the point due to incorrect length and that due to incorrect direction are equal, then the limiting length of the offset would be ?

$$A) \quad S = 10m, \quad x = l \sin \theta$$

$$\frac{x}{l} = \frac{1}{100}$$

$$l = \frac{1}{\sin \theta} \sqrt{(.025S)^2 - x^2}$$

$$l = \frac{1}{\sin \theta} \sqrt{(.025 \times 10)^2 - l^2 \sin^2 \theta}$$

$$(l \sin \theta)^2 = .0625 - (l \sin \theta)^2$$

$$2l \sin \theta = .0625$$

A) Error due to incorrect direction = $\sin \theta$

$$\frac{l}{100} = \sin \theta$$

$$l = \frac{1}{\sin \theta} \sqrt{(0.025)^2 - x^2}$$

$$l = \frac{1}{\sin \theta} \sqrt{0.0625 - \left(\frac{l}{100}\right)^2}$$

$$l = \frac{100}{100} \sqrt{0.0625 - \left(\frac{l}{100}\right)^2}$$

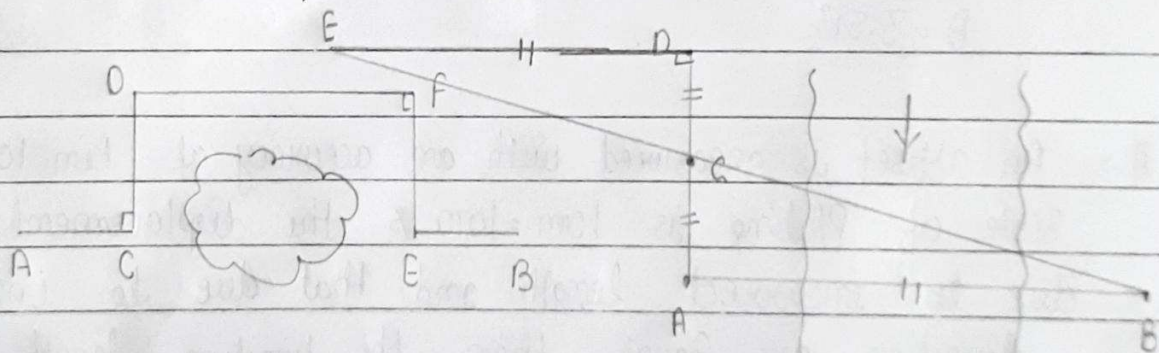
$$\left(\frac{100l}{100}\right)^2 + \left(\frac{l}{100}\right)^2 = 0.0625$$

$$\left(\frac{9l}{100}\right)^2 = 0.0625$$

$$l = 17.67$$

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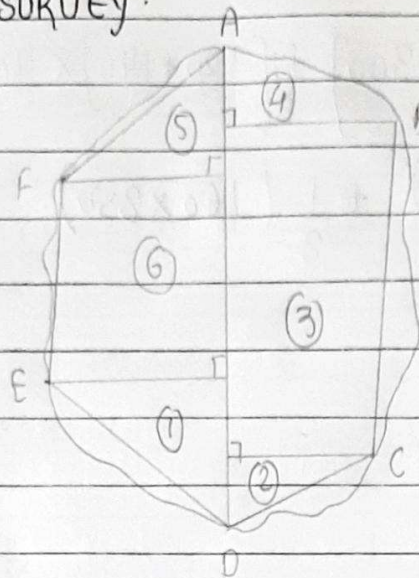
OBSTACLES IN CHAINING



Chaining Round the obstacle
Possible

Chaining round the
obstacle not possible

CROSS - STAFF SURVEY.

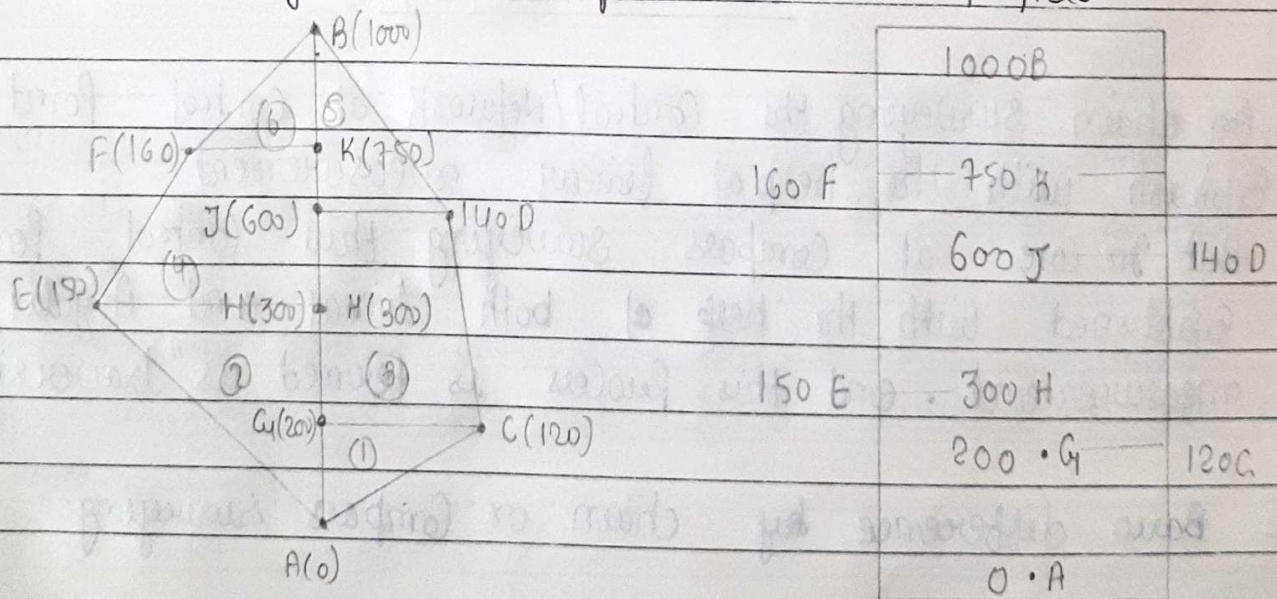


$$A = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$$

$$A = \sum A_i$$

- Cross staff survey is a special type of chain survey to locate the boundaries of a field for calculating its area.
- In this survey a base line is selected through the middle portion of the field and the entire area is divided into number of simple regular figures like triangular and trapezium.
- In this method cross staff is used for setting out the perpendicular to base line.

Q.1) Show the cross staff survey a field ABCDEF from the field book measurement given and compute the area of field



$$A = \sum A$$

$$= \left(\frac{1}{2} \times 120 \times 200 \right) + \left[\frac{1}{2} \times 150 \times 300 \right] + \left[\frac{1}{2} (120 + 140) \times 400 \right] + \frac{1}{2} (150 + 160) \times 350$$

$$+ \frac{1}{2} (140 \times 400) + \frac{1}{2} (160 \times 250)$$

$$A = 204250 \text{ m}^2$$

or

Area No.	chainage	Baselength(m)	Offset(m)	mean Offset(m)	Area(m ²)
1	0-200	200	120	60	12000
2	0-300	300	150	75	22500
3	200-600	400	120-140	130	52000
4	300-750	450	150-160	155	69750
5	600-1000	400	140	75	28000
6	750-1000	250	160	80	20000

$$\Sigma = 204250 \text{ m}^2$$

COMPASS SURVEY

- In chain surveying the Control/Network of Control Point is established with the help of linear measurement
- But in case of Compass surveying these Control Points are established with the help of both linear and Angular measurement. and this process is termed as "traversing"
- Basic difference b/w chain or Compass surveying.

CHAIN SURVEYING

- It requires only linear measurement
- Here framework consist of network of triangle
- Here check lines are required to check the accuracy
- It is suitable to be use for small area, flat area where accuracy ~~is~~ required is less

COMASS SURVEYING

- It require both linear and angular measurement
- Here framework consist of connected lines forming a polygon
- ~~Here~~ there accuracy is checked by method of adjustment
- It is suitable for large area where accuracy required is more.

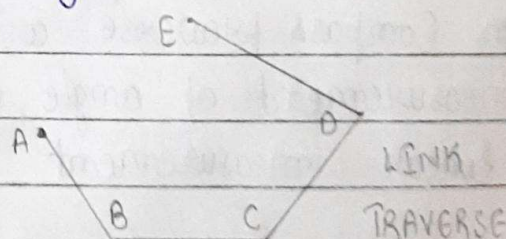
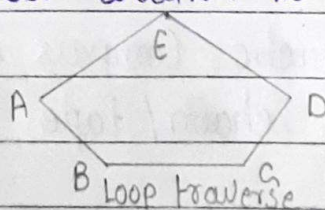
Note: Here framework consist of number of connected lines forming open or closed polygon according to which it is further classifies

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(a) CLOSED TRAVERSE

A closed traverse start from a station and closes either on another station whose location is already known.

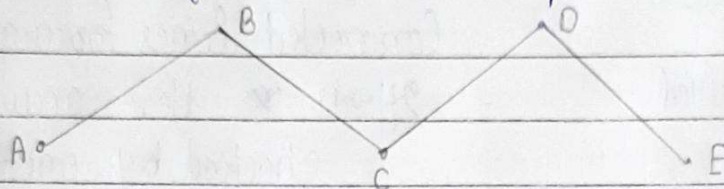
Eg:



- If traverse start & ends at the same point whose location is known this type of closed traverse is termed as loop traverse.
- If the traverses starts and ends at different points of known location this type of closed traverse is termed as link traverse
- In case of close traverse, it is easy to detect the errors and adjust or balance it.

(B) OPEN TRAVERSE

- An open traverse is that where start station is known or establish, but the station at which the traverse is completed/closes is not known or establish



- It is used for the survey of roads, railways, canals, tunnels etc.
- An open traverse cannot be properly checked, and adjusted or balanced hence it is avoided

- Traversing can be done by any of following method

(A) COMPASS TRAVERSE

(B) PLANE TABLE TRAVERSE

(C) THEODOLITE TRAVERSE

(D) STADIA TRAVERSE

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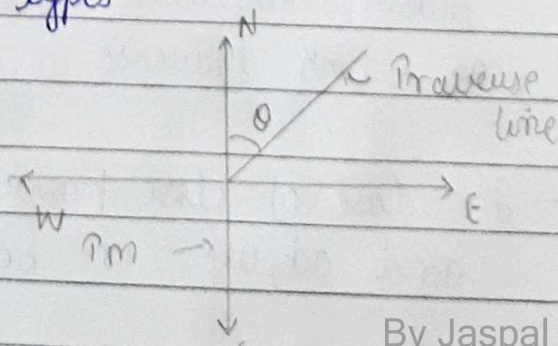
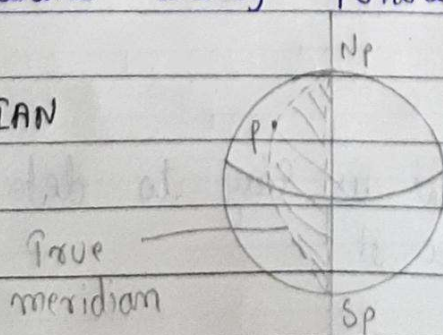
COMPASS TRAVERSE

- In Compass traverse a magnetic compass is used for measurement of angle and chain/tape is used for linear measurement

MEASUREMENT OF ANGLE

- The direction of the traverse line is defined by the horizontal angle which this line makes with the reference line that is termed as "Meridian"
- These Meridians are of following types

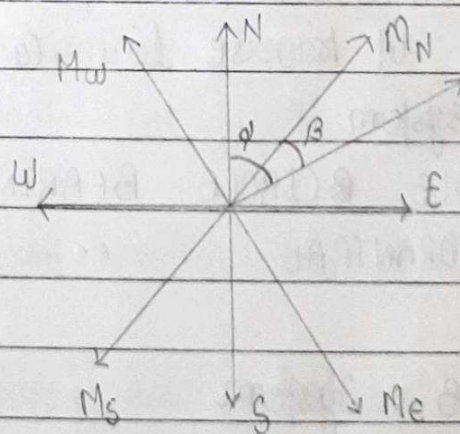
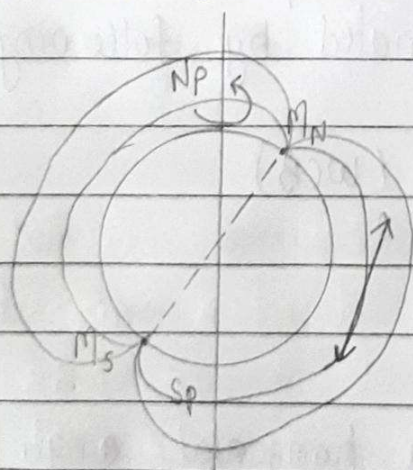
(A) TRUE MERIDIAN



- A true Meridian Passing through a Point on the Earth surface is a line which is obtained by the intersection of Earth surface with the plane passing through that point and the geographical north & south pole.
- True meridian can also be determined by astronomical observation
- TM at a point is always fixed
- TM through various points/stations are not parallel to each other but will converge at the poles, However for small survey these are considered to be parallel to each other.

MAGNETIC MERIDIAN

- Magnetic meridian at a point is direction indicated by a freely suspended balanced magnetic needle bar
- It indicates the direction of magnetic north and magnetic south
- Magnetic poles are not fixed hence Magnetic Meridian is also not fixed and changes with time



GRID MERIDIAN

- For survey of a Country, the true meridian passing through the Central place is sometimes used as reference meridian for entire Country and termed as grid meridian.

- The meridian at all other places is considered to be parallel to this grid meridian

ARBITRARY MERIDIAN

- The meridian which is taken in any convenient arbitrary direction is termed as arbitrary direction
Eg- chimney, church, tower, railway station.

- The horizontal angle made by the traverse line with the reference line / meridian is termed as bearing.
- Depending upon the reference line selected these bearing are of following type

(A) TRUE BEARING / AZIMUTH

(B) MAGNETIC BEARING

(C) GRID BEARING

(D) ARBITRARY BEARING

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REPRESENTATION / DESIGNATION OF BEARING

- Bearing of traverse line can be designated by following two system

(a) WHOLE CIRCLE BEARING SYSTEM (WCB)

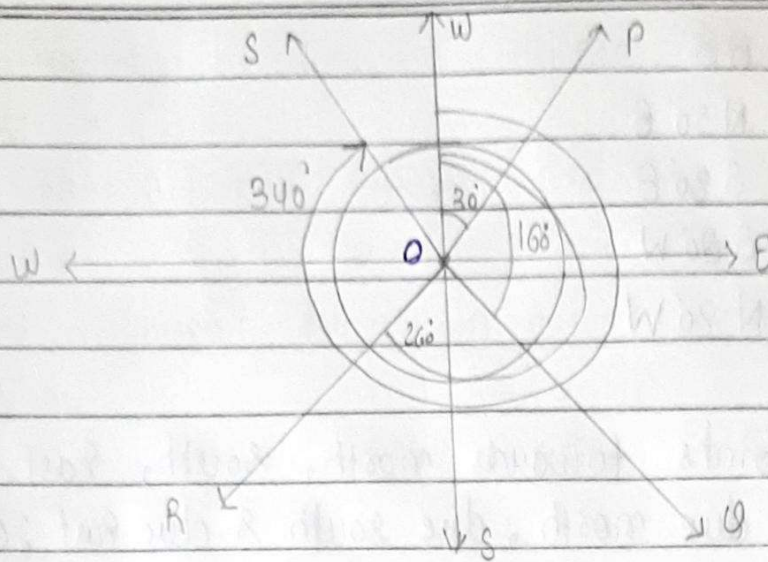
(b) QUADRANTAL

ee

ee

(i) WCB System

- In this system bearing of a line is the horizontal angle b/w the line and north end of the reference meridian
- It is measured in clockwise direction
- The whole circle bearing line varies from 0-360
- It is also termed as AZIMUTH as reference is true north
- It is measured with the help of PRISMATIC COMPASS



Note: when the WCB of line exceeds 90° , & if it is reduced to corresponding angle less than 90° then it is termed as "REDUCED BEARING" (RB)

CASE	WCB RANGE	RB	QUADRANT	EXAMPLE
1	$0^\circ - 90^\circ$	WCB	NE	OP : 30° NE
2	$90^\circ - 180^\circ$	$180^\circ - \text{WCB}$	SE	OQ : 20° SE
3	$180^\circ - 270^\circ$	$\text{WCB} - 180^\circ$	SW	OR : 80° SW
4	$270^\circ - 360^\circ$	$360^\circ - \text{WCB}$	NW	OS : 20° NW

(B) QUADRANTAL BEARING (QB) SYSTEM

- QB of a line is the acute angle which the line makes with the reference meridian
- It can be measured from north or south point whichever is closer
- It is measured both clockwise and anticlockwise direction
- It varies in range of $0^\circ - 90^\circ$

It is measured with the help of "Surveyor's Compass"

- QB is reported as follows

(i) first the letter N or S representing north or south point is mentioned

(ii) mention the angle of the bearing

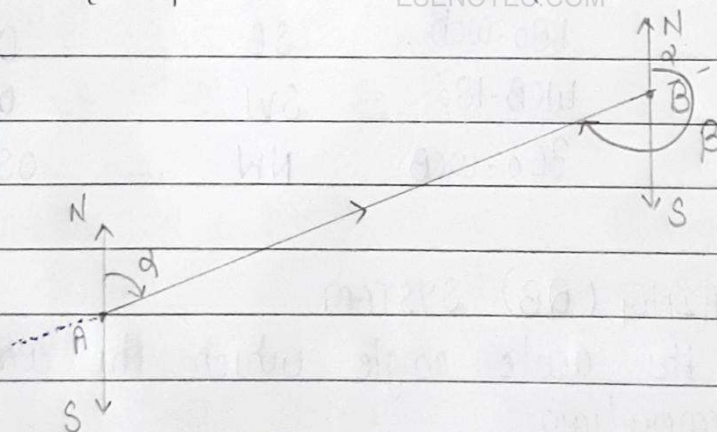
(iii) Lastly E or W representing East or West point is reported

Eq	LINE	QB
	OP	N 30° E
	OQ	S 20° E
	OR	S 80° W
	OS	N 20° W

Note: when a line points towards north, south, East, West it is reported as due north, due south & due East; due west respectively

Eq	QB	wcb
	S° : Due south	180°
	N° : Due north	0°
	N 90° W or S 90° W : Due west	270°
	N 90° E or S 90° E : Due east	90°

FORE BEARING (FB) AND BACK BEARING (BB)



LINE	FB	BB
AB	α	β
BA	β	α

- The bearing of the line in direction of the progress of survey is called Fore Bearing (FB) and in direction opposite to the progress of survey is called back bearing (BB)
- FB and BB of a line differ by an angle of 180°

(1) if $FB < 180^\circ$, $BB = FB + 180^\circ$

if $FB > 180^\circ$, $BB = FB - 180^\circ$

Hence if bearing of the line is reported in wcb system then $BB = FB \pm 180^\circ$

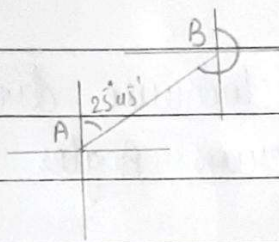
(iii) If bearing of line is reported in AB system then BB angle = FB Angle
But $N \rightarrow S$ and $E \rightarrow W$ and Vice Versa

Q. 1 FB of different lines are given. Compute Corresponding BB

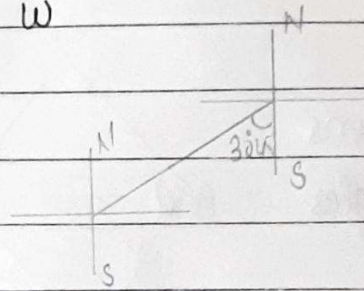
(a) $25^\circ 45'$ (c) $S 30^\circ 40' W$

(b) 350° (d) $N 35^\circ W$

Ans)



(ii)



(i) $FB = 25^\circ 45'$

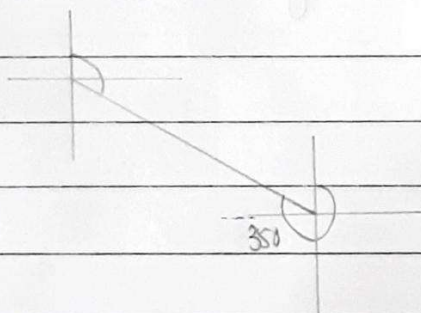
$BB = 180 + 25^\circ 45'$

$BB = 205^\circ 45'$

$FB = S 30^\circ 40' W$

$BB = N 30^\circ 40' W$

(iii)



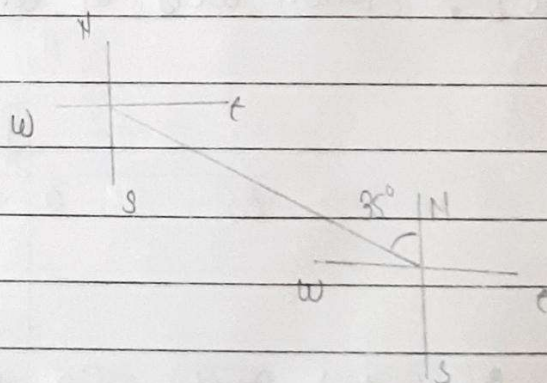
$FB = 350^\circ$

$BB = 90 + (350 - 270)$

$BB = 90 + 80^\circ$

$BB = 170^\circ$

(iv)



$FB = N 35^\circ W$

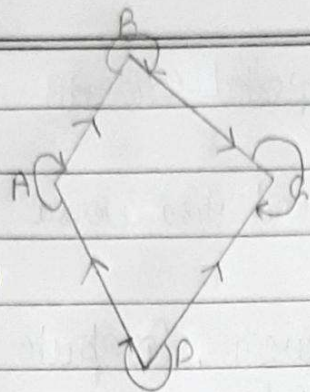
$BB = S 35^\circ E$

INCLUDED ANGLES

- When two lines meet at a point the angle enclosed between them is termed as Included Angle
- Included angle may either be Interior or Exterior
- In surveying Included Angle is defined as Angle measured in clockwise direction from Preceding line to forward line.

Case (i)

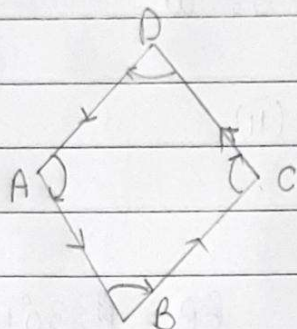
If traverse
Included Angle are



sums in clock-wise direction
Interior Angle

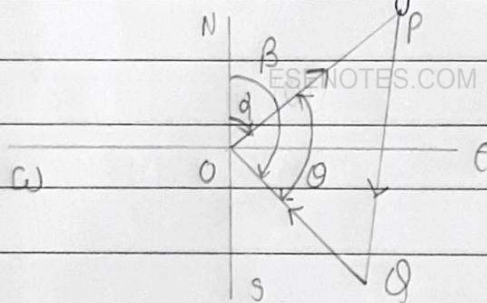
Case(ii)

If traverse runs
Included Angles



In Anti-clockwise direction
are Interior Angles

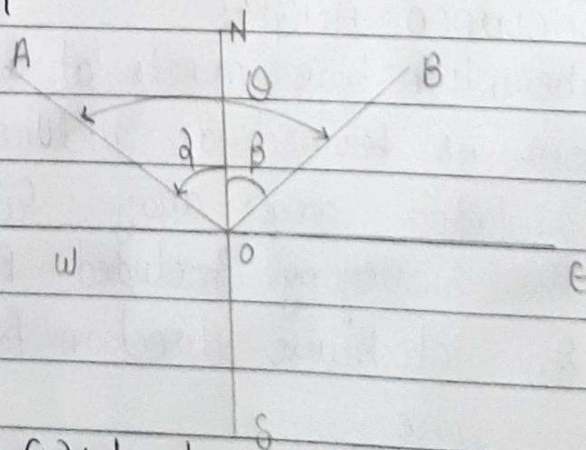
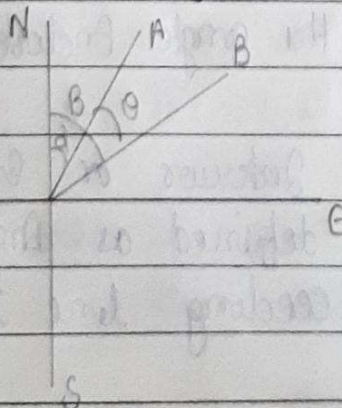
#*) Calculation of Included angles from bearing and Vice-Versa
Case I: When WB of lines are given



Included Angle = FB of Forward line - BB of Previous line
 $= \alpha - \beta$

Note: if $\alpha - \beta$ is (-)ve add 360° to get Included Angle

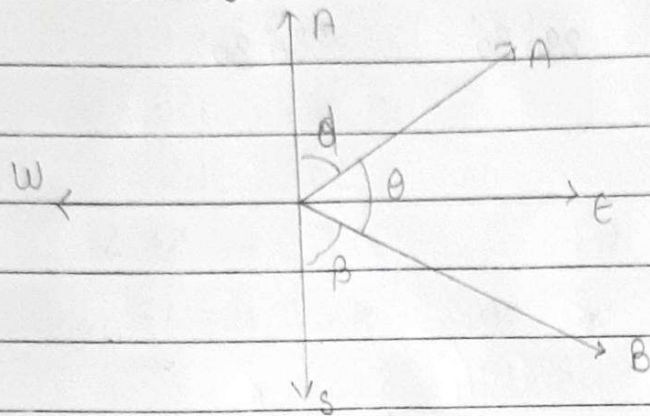
Case II: When AB of line are given



(a) When bearing is measured on same side
of Common meridian, $\theta = \beta - \alpha$

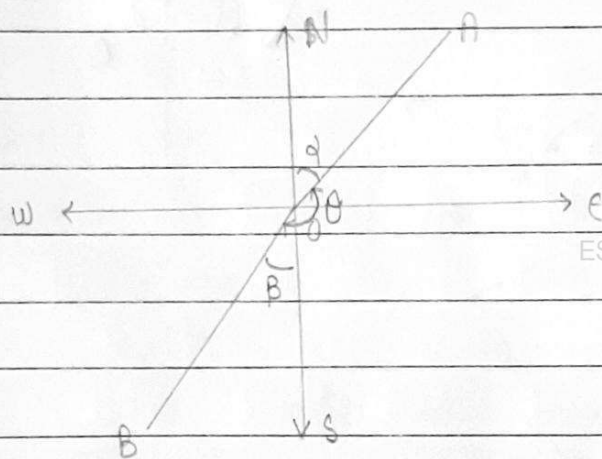
(b) When bearing are measured on
opposite side of Common meridian, $\theta = \alpha + \beta$

(c) when bearings are measured in same side of different meridians



$$\theta = 180^\circ - (\alpha + \beta)$$

(d) when bearing are measured in opposite direction on different sides of meridian.



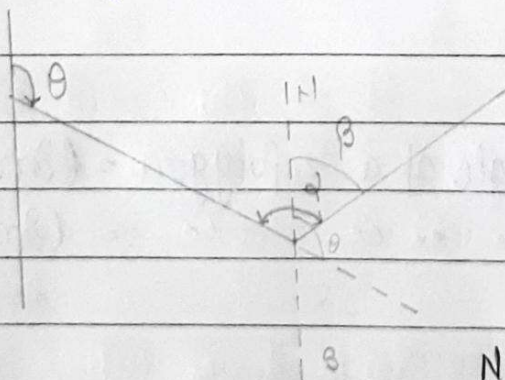
$$\theta = 90^\circ - \alpha + 90^\circ + \beta$$

$$\theta = 180^\circ + \beta - \alpha$$

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CALCULATION OF BEARING FROM INCLUDED ANGLE

- By knowing the included angle of traverse, bearing of a line, the bearing of other line can also be computed.



$$180^\circ = \theta + (\alpha - \beta)$$

$$180^\circ - \theta + \beta = \alpha$$

$$\beta = (\alpha + \theta) - 180^\circ$$

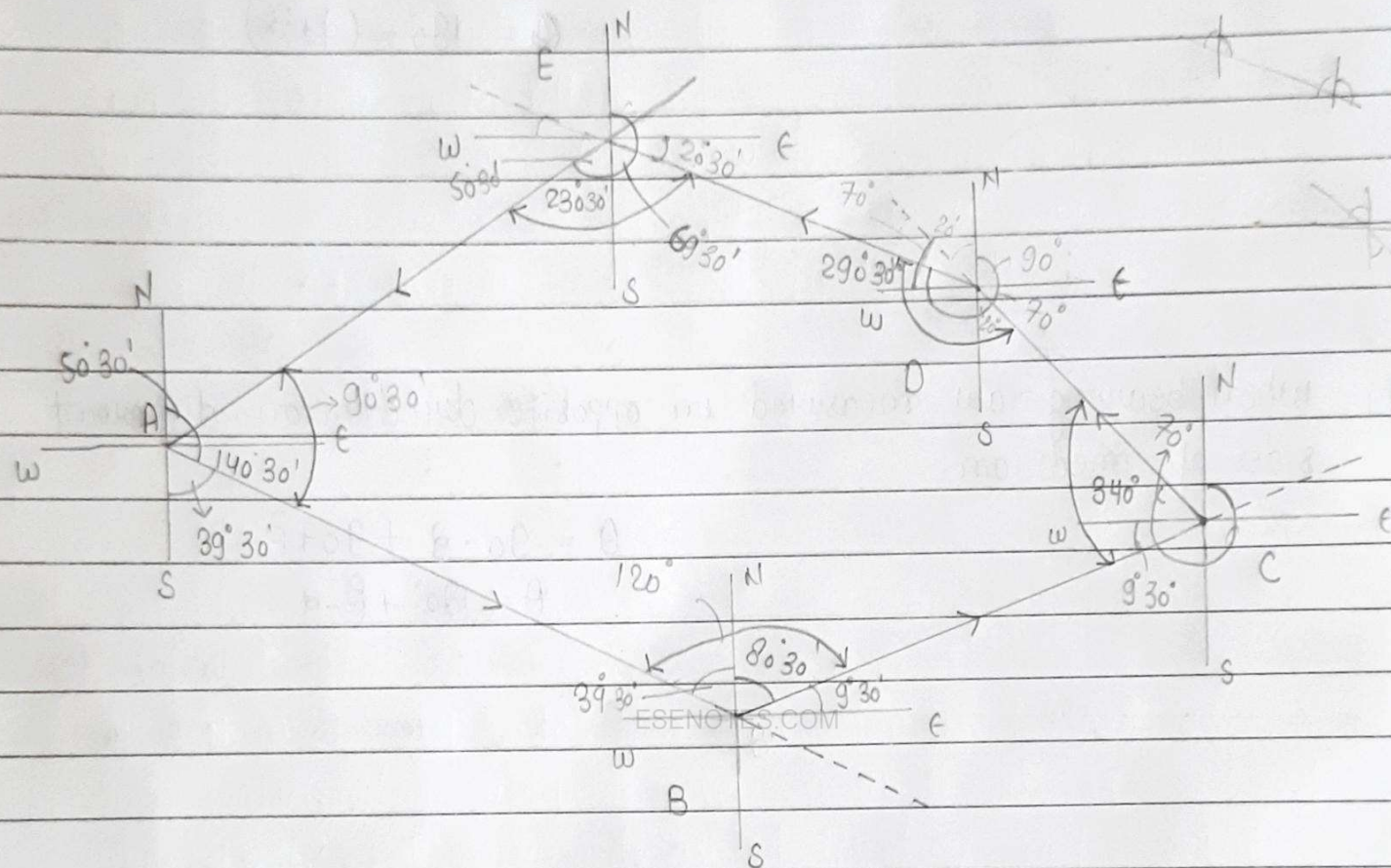
FB of next line = FB of prev. line + Included Angle

Note: If this sum is more than 180° ,
subtract 180° & if this sum is less than 180°
add 180°

Date.....

Q → The following bearings were observed for a closed traverse ABCDEA

LINE	AB	BC	CD	DE	EA
FB	$140^{\circ} 30'$	$80^{\circ} 30'$	$340^{\circ} 0'$	$290^{\circ} 30'$	$230^{\circ} 30'$



$$\angle A = 140^{\circ} 30' - 50^{\circ} 30' = 90^{\circ} 30'$$

$$\angle B = 80^{\circ} 30' + 39^{\circ} 30' = 120^{\circ}$$

$$\angle C = 70^{\circ} + 9^{\circ} 30' = 79^{\circ} 30'$$

$$\angle D = 20^{\circ} + 90^{\circ} + 20^{\circ} 30' = 130^{\circ} 30'$$

$$\angle E = 69^{\circ} 30' + 50^{\circ} 30' = 120^{\circ}$$

Note: Sum of all Interior Angle of a polygon $= (2n-4) \times 90^{\circ}$
 or Exterior $= (2n+4) \times 90^{\circ}$

Q. The following Angles were observed in clockwise direction in an open traverse. if the MB of line AB is $241^{\circ}30'$ what would be the bearing of other line

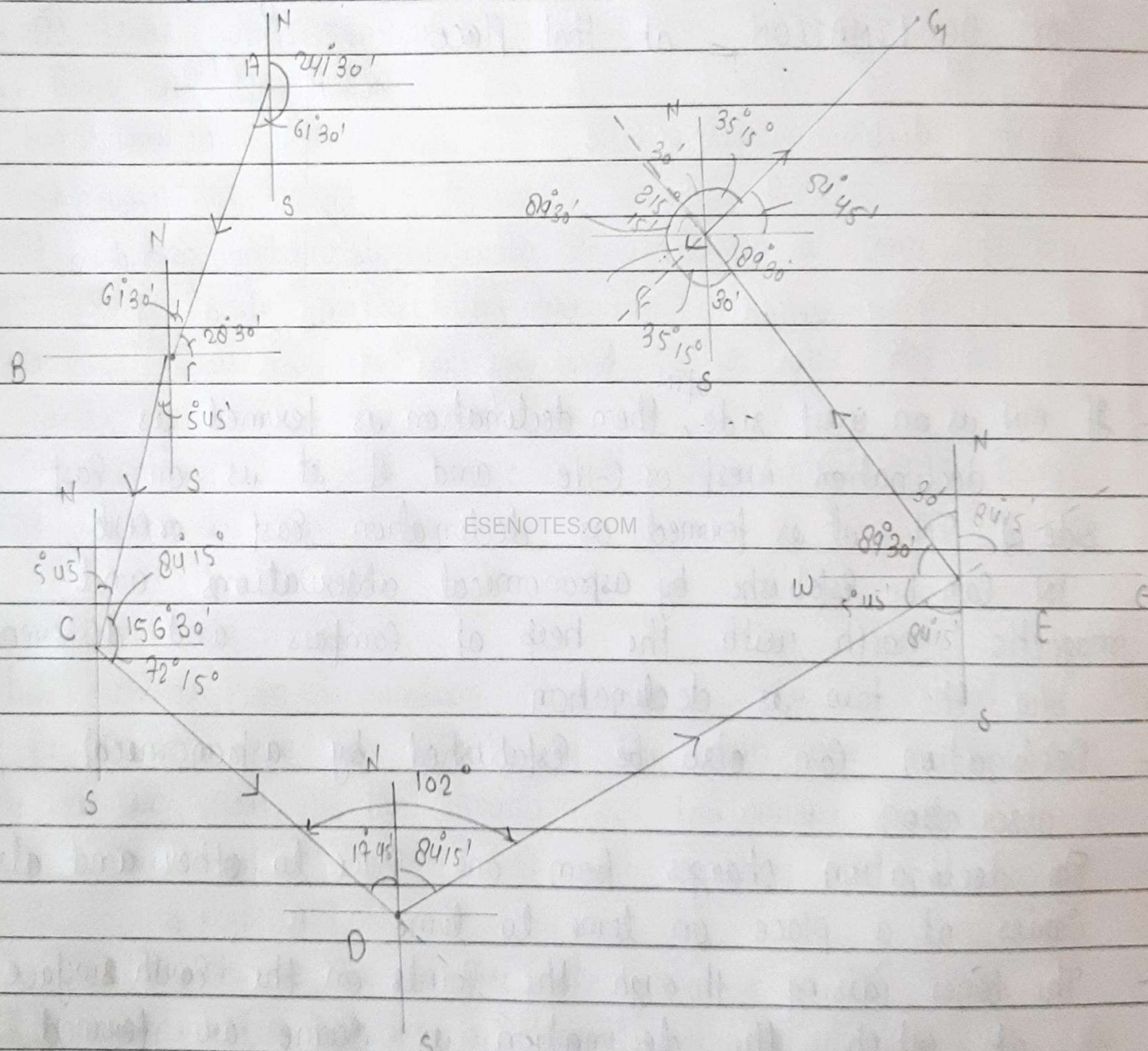
$$\angle ABC = 124^{\circ}15'$$

$$\angle BCD = 156^{\circ}30'$$

$$\angle CDE = 102^{\circ}$$

$$\angle DEF = 95^{\circ}15'$$

$$\angle EFG = 815^{\circ}45'$$



Line FB

AB $241^{\circ}30'$

BC $61^{\circ}30' + 124^{\circ}15' = 185^{\circ}45'$

CD $5^{\circ}45' + 156^{\circ}30' = 162^{\circ}15'$

DE $84^{\circ}15'$

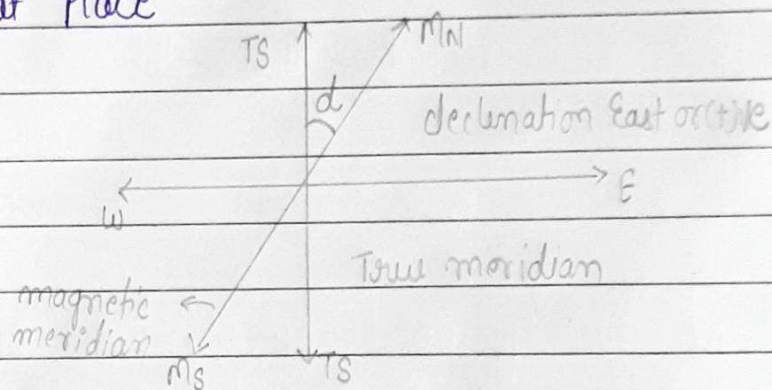
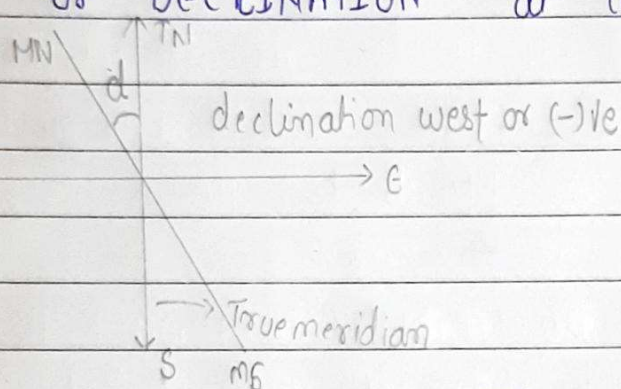
EF $360 - 30' = 359^{\circ}30'$

FG $35^{\circ}15'$

Note: If any angle comes out to be (-)ve add 360° in it

MAGNETIC DECLINATION

- In general magnetic meridian and True meridian do not coincide with each other.
- The horizontal angle between the magnetic meridian and True meridian is termed as magnetic declination or DECLINATION at that place



- If MN is on west side ^{of TN}, then declination is termed as declination west or (-)ve and if it is on East side of TN it is termed as declination East or (+)ve.
- TN can be established by astronomical observations and magnetic north with the help of Compass and difference b/w the two is declination
- Declination can also be established by astronomical observation
- The declination changes from one place to other and also varies at a place on time to time
- The lines passing through the points on the Earth surface at which the declination is same are termed as ~~iso~~ ISOGONIC LINES
- AGONIC LINES are those isogonic lines which pass through the point of zero declination (i.e. at these points TN and MN coincide)

VARIATION OF MAGNETIC DECLINATION

- Declination at a point is not constant and varies from time to time due to different reasons according to which variations are classified as follows.

(a) SECULAR VARIATION

(b) ANNUAL VARIATION

(c) DIURNAL VARIATION

(d) IRREGULAR VARIATION.

(i) SECULAR VARIATION

- It occurs continuously over a long period of time in b/w (100-350 yrs) to change the direction
- Its exact cause is not known and no reliable method of predicting it is also available
- The annual rate of change of this variation is not same and is in between 5-10 min

(ii) ANNUAL VARIATION

- It is a change in declination at a place over a period of 1 year.
- It is caused due to rotation of the Earth around the sun
- The annual rate of this variation is 1-2' min

(iii) DIURNAL VARIATION

- It is the change in the declination at a place in 24 hrs
- It is due to rotation of the Earth about its own axis
- The amount of variation in this case is fraction of a minute to over 12'
- It also depends upon following factors.
 - (i) Geographical position of the place (It is less near Equator & more near Poles)

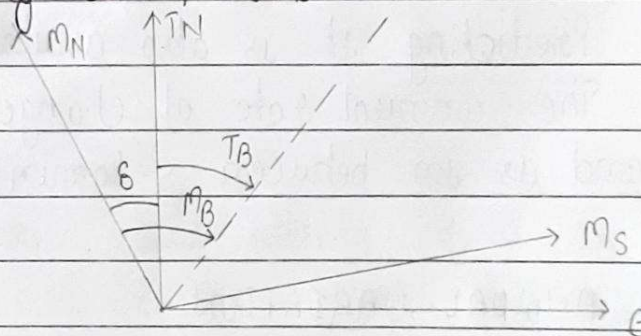
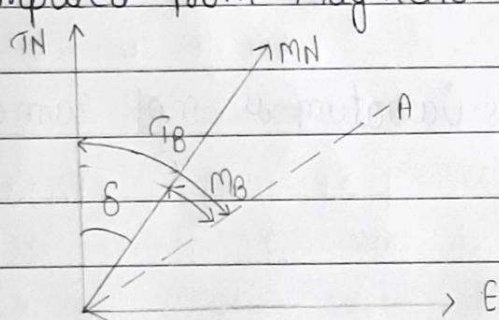
Date.....

- (c) Time of day (more in day & less in night)
 (d) Season of Year (more in summer & less in winter)
 (e) The Year of the cycle of secular variation

(10) IRREGULAR VARIATION.

- Magnetic disturbances in the Earth magnetic field leads to the Irregular variation
- Such type of variation are uncertain, random & unpredictable
- It is due to natural phenomenon like Earthquake, volcanic eruption, Tsunamis
- It may extend upto 2°

IF MAGNETIC DECLINATION is known, True bearing can be computed from Magnetic bearing as follows



if declination is East

$$TB = MB + \delta$$

$$MB = TB - \delta$$

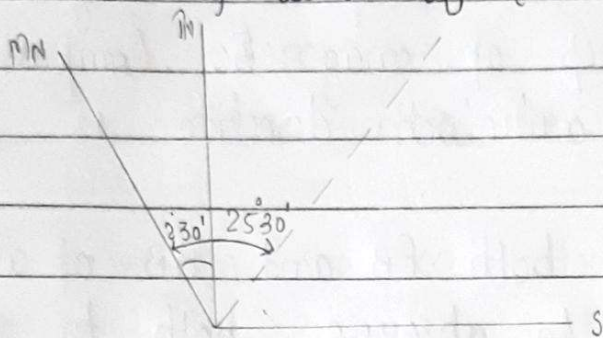
if declination is west

$$TB = MB - \delta$$

$$MB = TB + \delta$$

Note: The above analysis is valid, if bearing are in WCB system, if bearings are in quadrant system, then either convert it in WCB system and do the above analysis or find the relation for such quadrant b/w TB & MB

b) A line AB was drawn to have a MB of $25^{\circ}30'$ in an old map, at that time the declination was $2^{\circ}30'E$. Determine the MB of line if the present declination is $5^{\circ}30'W$

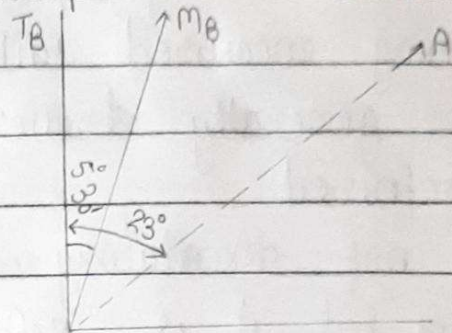


$$\delta = 2^{\circ}30'E, MB = 25^{\circ}30'$$

$$TB = MB - \delta$$

$$TB = 25^{\circ}30' - 2^{\circ}30'$$

$$TB = 23^{\circ}$$



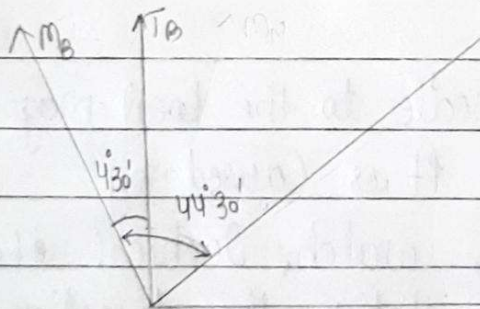
$$TB = MB + \delta$$

$$MB = TB - \delta$$

$$MB = 23^{\circ} - 5^{\circ}30'$$

$$MB = 17^{\circ}30'$$

a) A line AB had the MB $44^{\circ}30'$ in 1910 when the declination was $4^{\circ}30'W$. Determine the MB of the same line in 1990 if the Annual declination change is observed as $6'$ Eastward.



$$TB = S + MB$$

$$= 4^{\circ}30' + 44^{\circ}30'$$

$$TB = 49^{\circ}$$

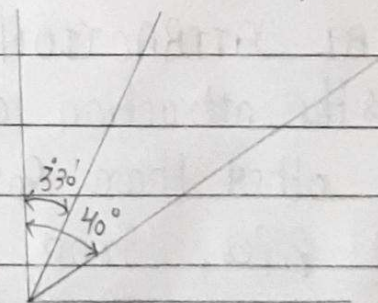
$$\text{change} = 6' \times 80$$

$$= 8'$$

$$\Rightarrow 4^{\circ}30' - 8' = 3^{\circ}30'W$$

$$TB = 44^{\circ}30' - 4^{\circ}30'$$

$$TB = 40^{\circ}$$



$$MB = 3^{\circ}30' + 49^{\circ}$$

$$MB = 52^{\circ}30'$$

$$MB = 40 - 3^{\circ}30'$$

$$MB = 36^{\circ}30'$$

Running a Compass Traverse

- In this traverse linear measurement of traverse line is done with the help of magnetic chain/tape and angles are being measured with the help of magnetic Compass.
- Here generally traverse in anticlockwise direction is preferred.
- It is not mandatory to observe both FB and BB of a line but it is recommended to observe both to identify the errors in traverse or presence of local attraction.
- In closed traverse sum of Internal Angle observed must be equal to, sum of theoretical sum of Internal Angles $(2N-4) \times 90^\circ$.
- If it is not same, then correction is applied equally to all the angles.

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LOCAL ATTRACTION

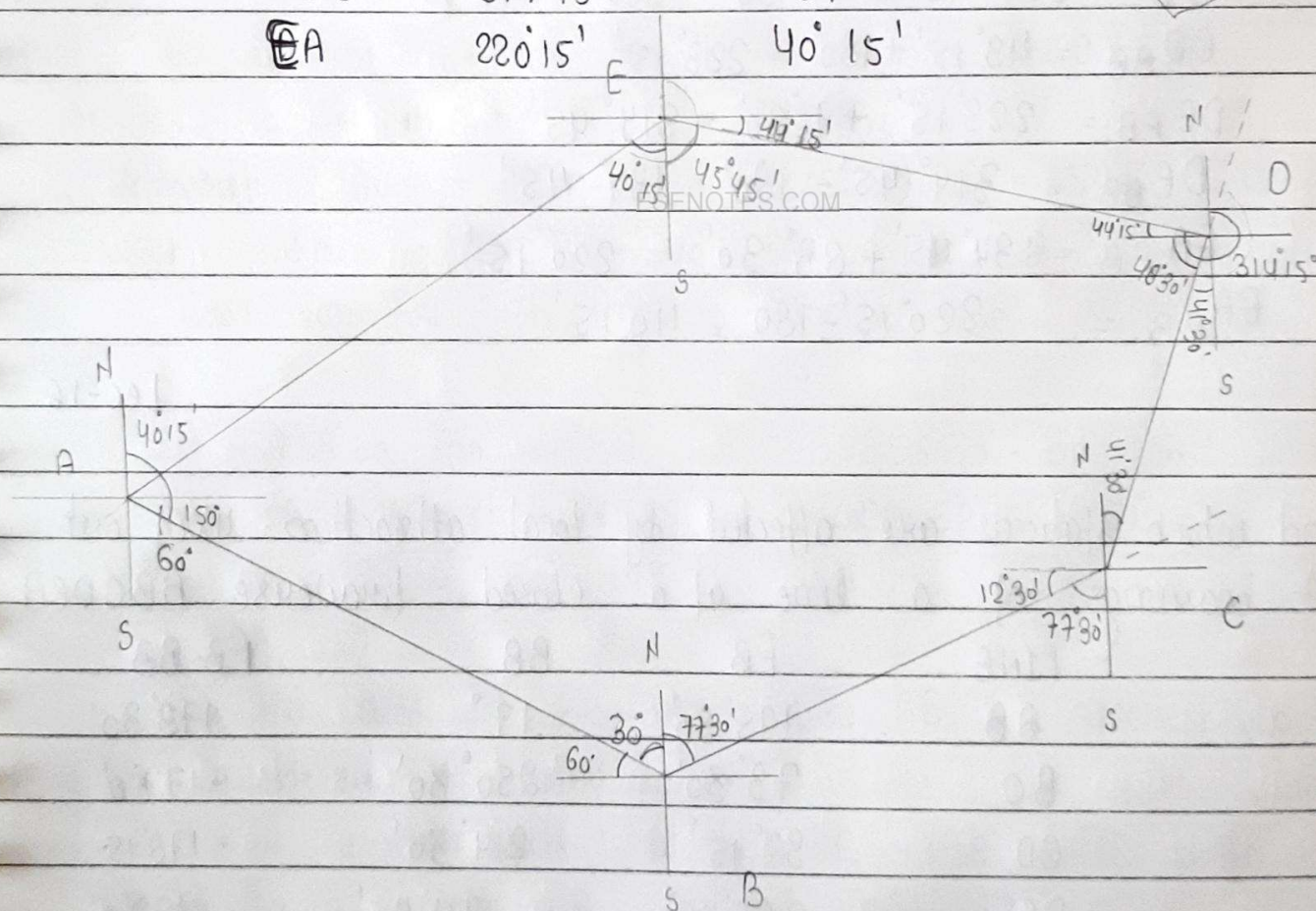
- It is the attraction of the magnetic needle to the local magnetic field other than Earth's magnetic field. It is caused by steel pipes, iron fences, electric wires, watch, technical etc.
- A freely suspended magnetic needle takes the direction of the Earth magnetic field only if there is no local attraction present in the area.
- The magnetic needle deviates from the magnetic meridian under local magnetic forces thereby affecting the magnetic bearing of the line.
- Local attraction can be identified by noting the FB and BB of a line.
- If the diff b/w FB & BB of a line is not 180° in that case either there is observational error in FB or BB or both or there is local attraction.

at one or both station of that line.

- Correction of the local attraction can be done by solving this Q.

Q) A Compass traverse ABCDEA was run anticlockwise and following bearings were taken, where local attraction was suspected. Determine the local attraction & correct the bearing.

LINE	FB	BB
AB	150°	$329^\circ 45'$
BC	$77^\circ 30'$	256°
CD	$41^\circ 30'$	$222^\circ 45'$
DE	$314^\circ 15'$	$134^\circ 45'$
EA	$220^\circ 15'$	$40^\circ 15'$



$$\angle A = 150^\circ - 40^\circ 15' = 109^\circ 45'$$

$$\angle B = 360 - [329^\circ 45' - 77^\circ 30'] = 107^\circ 45'$$

$$\angle C = 360^\circ - [256^\circ - 41^\circ 30'] = 145^\circ 30'$$

$$\angle D = 314^\circ 15' - 222^\circ 45' = 91^\circ 30'$$

$$\angle E = 220^\circ 15' - 134^\circ 45' = 85^\circ 30'$$

Date.....

observed $\sum \text{Angles} = \angle A + \angle B + \angle C + \angle D + \angle E = 540$

theoretical $\sum (2N - 4)90^\circ = 540$

There is no error in summation of internal angle

Since line EA = FB - BB = 180° , station E and A are free from local attraction

$AB_{FB} = 150^\circ$, $EA_{BB} =$

$AB_{BB} = 150 + 180 = 330^\circ$

$AB_{BB} \text{ observed} = 329^\circ 30'$ hence local attraction = $329^\circ 45' - 330^\circ = -15'$

$BC_{FB} = 330^\circ + 107^\circ 45' - 360^\circ = 77^\circ 45'$

$BC_{BB} = 77^\circ 45' + 180^\circ = 257^\circ 45'$

$CD_{FB} = 257^\circ 45' + 145^\circ 30' - 360^\circ = 43^\circ 15'$

$CD_{BB} = 43^\circ 15' + 180^\circ = 223^\circ 15'$

$DE_{FB} = 223^\circ 15' + 91^\circ 30' = 314^\circ 45'$

$DE_{BB} = 314^\circ 45' - 180^\circ = 134^\circ 45'$

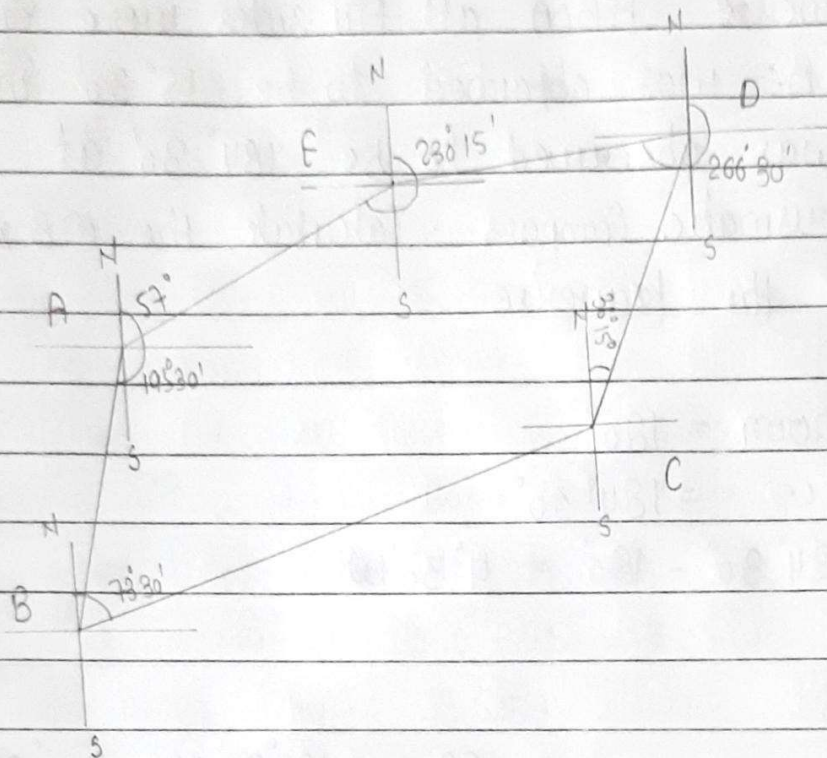
$EA_{FB} = 134^\circ 45' + 83^\circ 30' = 220^\circ 15'$

$EA_{BB} = 220^\circ 15' - 180^\circ = 40^\circ 15'$

Lec-16

Q. Find which stations are affected by local attraction. Work out correct bearings of a line of a closed traverse ABCDEA

LINE	FB	BB	FB-BB
AB	$195^\circ 30'$	17°	$178^\circ 30'$
BC	$73^\circ 30'$	$250^\circ 30'$	$-177^\circ 0'$
CD	$36^\circ 15'$	$214^\circ 30'$	$-178^\circ 15'$
DE	$266^\circ 45'$	$84^\circ 45'$	$182^\circ 0'$
EA	$234^\circ 15'$	57°	$179^\circ 15'$



$$\angle A = 195^\circ 30' - 57^\circ = 138^\circ 30'$$

$$\angle B = 73^\circ 30' - 17^\circ = 56^\circ 30'$$

$$\angle C = 360^\circ - 250^\circ 30' + 36^\circ 15' = 145^\circ 45'$$

$$\angle D = 266^\circ 30' - 84^\circ 15' = 182^\circ 15'$$

$$\angle E = 230^\circ 15' - 84^\circ 45' = 145^\circ 30'$$

$$\text{Sum of Included Angle} = \angle A + \angle B + \angle C + \angle D + \angle E \\ = 542^\circ 30'$$

$$\text{Theoretical Sum} = (2n-4)90^\circ = (2 \times 5 - 4)90^\circ = 540^\circ$$

$$\text{Error} = 542^\circ 30' - 540^\circ = 2^\circ 30'$$

$$\text{Correction for each Angle} = \frac{-2^\circ 30'}{5} = -30'$$

$$\angle A = 138^\circ 30' - 30' = 138^\circ$$

$$\angle B = 56^\circ 30' - 30' = 56^\circ$$

$$\angle C = 145^\circ 45' - 30' = 145^\circ 15'$$

$$\angle D = 182^\circ 15' - 30' = 180^\circ$$

$$\angle E = 145^\circ 30' - 30' = 145^\circ$$

Since the diff in FB & BB of line AB is nearest to 180° , it is considered for computation

$$\text{Error in diff of FB \& BB} = 178^\circ 30' - 180^\circ = -1^\circ 30'$$

$$\text{Hence Correct FB} = 195^\circ 30' + 45' = 196^\circ 15'$$

$$\text{ee ee ee} = 17^\circ - 45' = 16^\circ 15'$$

$$BC_{FB} = 16^\circ 15' + 56^\circ = 72^\circ 15'$$

$$BC_{BB} = 72^\circ 15' + 180^\circ = 252^\circ 15'$$

$$CD_{FB} = 252^\circ 15' + 145^\circ 15' - 360^\circ = 37^\circ 30'$$

$$CD_{BB} = 37^\circ 30' + 180^\circ = 217^\circ 30'$$

$$DE_{FB} = 217^\circ 30' + 51^\circ 45' = 269^\circ 15'$$

$$DE_{BB} = 269^\circ 15' - 180^\circ = 89^\circ 15'$$

$$EA_{FB} = 89^\circ 15' + 145^\circ = 234^\circ 15'$$

$$EA_{BB} = 234^\circ 15' - 180^\circ = 54^\circ 15'$$

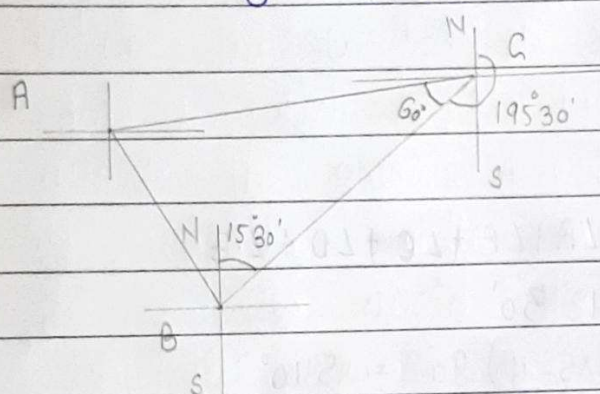
- a) In an anticlockwise traverse ABCA all the sides were equal. Magnetic fore bearing of BC was obtained to be $15^\circ 30'$. The bearing of sun was observed to be $184^\circ 30'$ at local noon with a prismatic compass. Calculate the MB & TB of all the sides of the traverse.

Solⁿ

$$TB \text{ of sun at noon} = 180^\circ$$

$$MB \text{ of sun at noon} = 184^\circ 30'$$

$$\delta = MB - TB = 184^\circ 30' - 180^\circ = 4^\circ 30' W$$



$$CA_{FB} = 195^\circ 30' + 60 = 255^\circ 30'$$

$$CA_{BB} = 255^\circ 30' - 180 = 75^\circ 30'$$

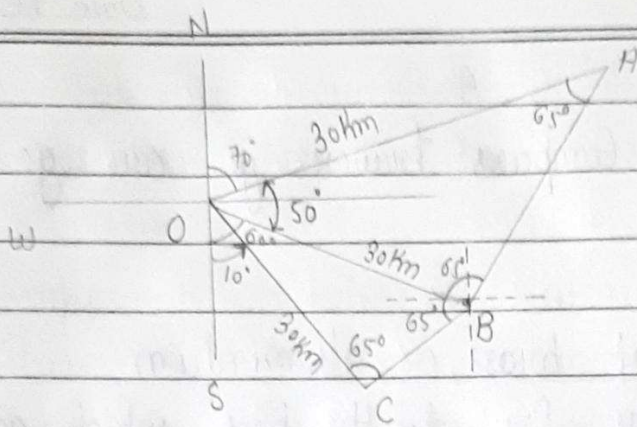
$$AB_{FB} = 75^\circ 30' + 60 = 135^\circ 30'$$

$$BC_{BB} = 15^\circ 30' + 180 = 195^\circ 30'$$

$$AB_{BB} = 135^\circ 30' + 180 = 315^\circ 30'$$

LINE	MB		TB	
	FB	BB	FB	BB
AB	$135^\circ 30'$	$315^\circ 30'$	131°	311°
BC	$15^\circ 30'$	$195^\circ 30'$	11°	191°
CA	$255^\circ 30'$	$75^\circ 30'$	251°	71°

- b) Three ships A, B, C started sailing at the same time from Goa. The speed of all the three ships was same equal to 30 km/hr. Their bearing were measured to be $N 70^\circ E$, $S 60^\circ E$, $S 16^\circ E$ respectively. After an hour the Captain of ship B determines the bearing of other 2 ships with respect to his own ship. After that he found out the distances. Calculate the value of bearings and distances which might have determined by the Captain.



$$\angle COB = 60^\circ - 10^\circ = 50^\circ$$

$$\angle OCB = \angle OBC = \frac{180 - 50}{2} = 65^\circ$$

$$CB = 2 \times 30 \cos 65^\circ$$

$$CB = 25.95 \text{ km}$$

$$\angle BOB = 180^\circ - 60^\circ = 120^\circ$$

$$\angle BOB = 120^\circ + 180^\circ = 300^\circ$$

$$\angle BOB = 300^\circ - 65^\circ = 235^\circ$$

$$\angle BOB = 55^\circ \text{ W}$$

$$\angle AOB = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$$

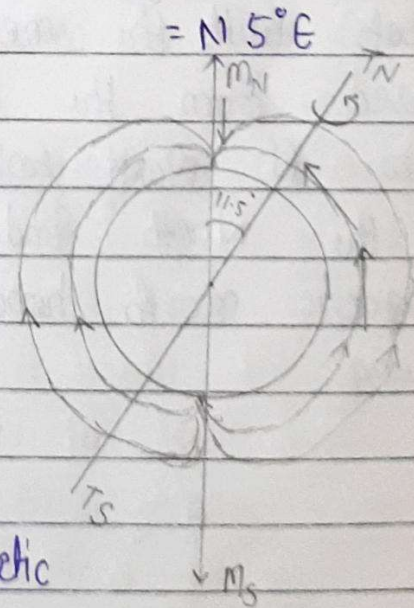
$$\angle OBA = \angle BAO = \frac{180 - 50}{2} = 65^\circ$$

$$AB = 2 \times 30 \cos 65^\circ = 25.95 \text{ km}$$

$$\angle BAO = 90^\circ + 65^\circ = 155^\circ > 180^\circ, \text{ hence } \angle BAO = 5^\circ$$

DIP

As Earth has its own Magnetic field, lines of forces are created by this & are directed towards the north and south magnetic poles



- A freely suspended magnetic needle align itself with the lines of magnetic forces of the Earth.

- The angle made by the lines of magnetic field with the Earth surface is called "DIP"

- Dip is zero at Equator and 90° at Poles

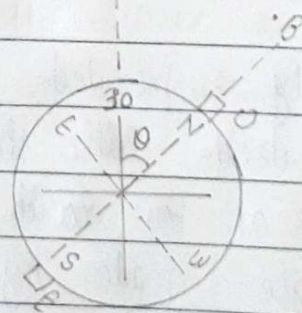
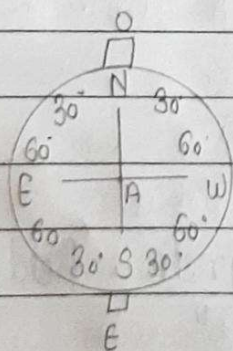
- At other places the value of dip varies b/wⁿ (0-90°)

TYPES OF COMPASS

- Magnetic Compass used in Compass traversing are generally of two types

(a) SURVEYOR'S COMPASS

- It consists of circular box of brass or aluminium
- The graduated ring is directly fixed to the box which governs the size of the compass normally that is in range of 10-15cm but can extend upto (5cm-20cm)
- The graduations are marked in degree and half degree
- When the compass is not in use object vane is folded which lifts the needle above pivot point thereby avoids the unnecessary wear and tear of pivot.
- The magnetic needle is made up of magnetic steel.
- In order to avoid the tip of the needle a small metallic rider is provided over the needle which is capable of sliding over it
- It is used to measure Quadrantal bearing
- Since the graduated rings turn with the sight vane not with the needle, the direction E and W are reversed from the actual
- The N mark rotates as the line of sight is rotated, but the North end of needle remains aligned to the magnetic north, hence directly giving the Quadrantal bearing



- Prior to the observation of bearings adjustment of the Compass is to be done which are of following types

(a) Temporary adjustment / station adjustment

- These are adjustment which are made at every set up of the instrument for example

(i) Centering : It is the process of setting up of the instrument over the station peg, by adjusting the legs of tripod and by centering the plumb bob over the station peg.

(ii) Levelling : Levelling is required so that the graduated ring is in horizontal plane and swings freely about the pivot point and must be in line of needle.

- It is done with the help of levelling screws or ball & socket arrangement

(b) Permanent adjustment

These adjustment once made generally last for a long period of time and are not required to be carried out for every station.

- They are needed for proper functioning of different components of the instruments for eg.

(i) Adjustment of the level

(ii) Adjustment of sight vane

(iii) Adjustment of sensitivity of the needle

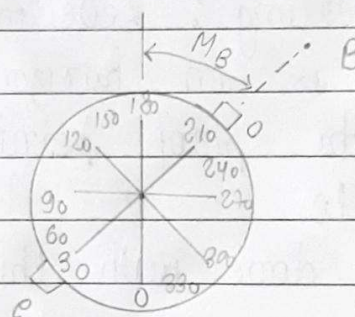
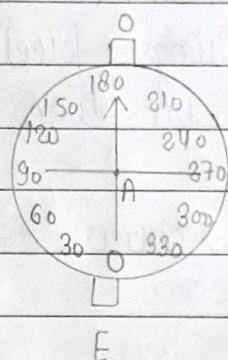
(iv) ee ee straightness of the needle

(v) ee ee Pivot.

(B) PRISMATIC COMPASS

- In this prism is used for taking the observation, thereby it is termed as PRISMATIC COMPASS.

- Its size is comparatively smaller than surveyor's Compass (25-100mm)
- Here magnetic needle used is comparatively broad.
- The ring is graduated in degree's and half degree's and is attached with needle & not with vane as in case of Surveyor's Compass
- Here graduation increases in clockwise direction with zero coinciding with the South end of the needle
- It is used for determination of whole circle bearing
- Here simultaneous sighting of the object and noting of reading from the ring is carried out



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- Temporary adjustment of Prismatic Compass Includes
 - 1) Levelling
 - 2) Centering
 - 3) focusing : In this case Prism is moved up and down up to an extent graduation on ring are clearly visible
- Permanent adjustment of Prismatic Compass Includes
 - 1) Adjustment of sensitivity of the needle

THEODOLITE

It is an instrument used to measure both horizontal and vertical angle.

- It can also be used for different surveying operation like levelling, extension of line, measurement of distances,

- Theodolite can be classified as follows

(a) TRANSIT THEODOLITE

- It is the type of theodolite in which telescope can be rotated ~~exactly~~ through an angle of 180° in vertical plane along the horizontal axis

- And if it cannot be rotated it is termed as non "transit theodolite"

- VERNIER THEODOLITE: Here Vernier's are used for taking the reading

- Here least count is $20''$

(d) PRECISE OPTICAL THEODOLITE

- It is fitted with precise optical system, and here readings are taken with the help of micrometer.

- Here least count is $1''$ or less

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Lec-18

TRAVERSING

- It is the process of measuring length and direction of survey line (Traverse line) in the field

- And traverse is a series of connected lines whose length and direction are being measured in the field.

- The field work for theodolite traverse includes

(a) Reconnaissance

(b) selection and Marking of survey line station

(c) Running and measuring traverse line

(d) Angular measurement and marking with offset

- Traverse are of following types

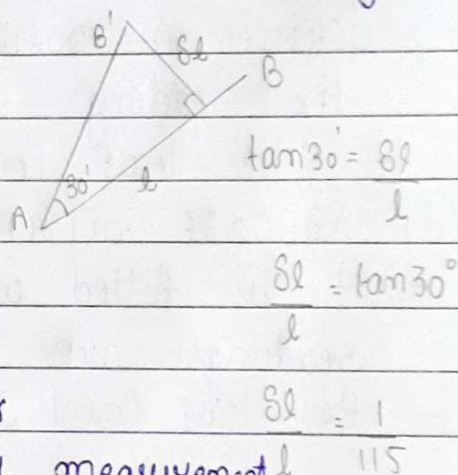
(a) OPEN TRAVERSE

(b) CLOSE TRAVERSE

- In traversing linear measurement are taken by
 - (a) chaining or Taping
 - (b) Tacheometric mtd.
 - (c) EDM

- The mtd of linear measurement adopted depends upon accuracy required in field work.
- The degree of accuracy obtained in linear and Angular measurement must be same.

Eg: If Compass has a least Count of $30'$, so the linear measurement must be done with an ^{Permissible} error or not more than 1 cm in 115



Note: If θ is small, $\theta \approx \tan \theta$ then for Consistent Precision in angular & linear measurement

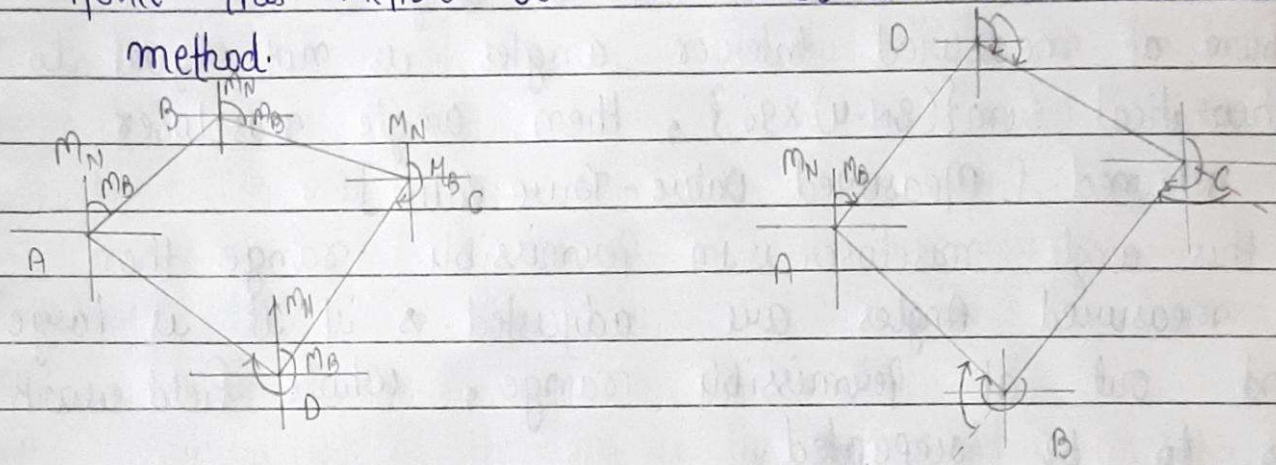
$$\boxed{\theta = \frac{Sl}{l}}$$

- In theodolite traversing Angular measurement is done with help of theodolite in any of the following ways.
 - (a) LOOSE / FREE NEEDLE METHOD OF BEARING
 - In this method direction of magnetic meridian is established at each traverse station and the direction of the line is measured with reference to magnetic meridian
 - A theodolite fitted with magnetic compass is used in this case to measure the magnetic bearing.

(B) FAST NEEDLE MTD

In this method magnetic meridian is established only at starting station and magnetic bearing of first line is noted or measured.

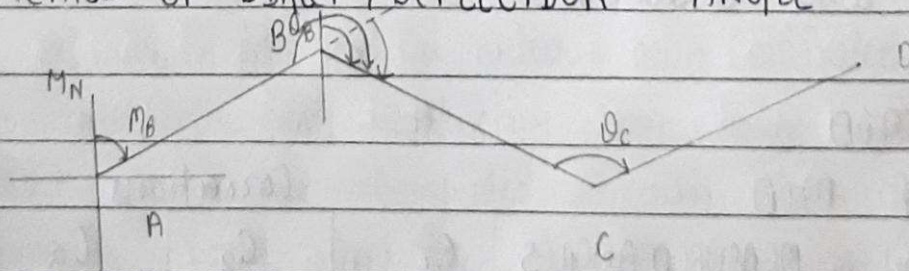
- The MB of remaining traverse line is measured with the magnetic bearing of the first line
- The MB of the first line has the accuracy of Compass and the MB of remaining lines has the accuracy of theodolite.
- Hence this method is more accurate than loose needle method.



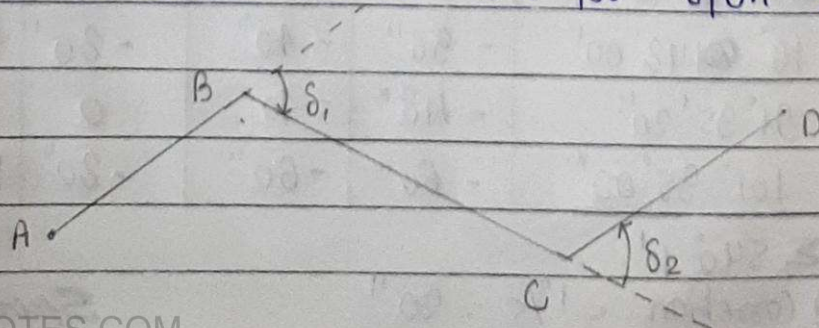
(C) METHOD OF INCLUDED ANGLE

- It is the most commonly used mtd of traversing.
- In this MB of initial line is measured in the field along with all included angles
- Bearing of all the remaining lines is determined from the bearing of initial line and the included lines

(d) METHOD OF DIRECT / DEFLECTION ANGLE



- Direct Angle mtd is similar to mtd of Included Angles
- This is suitable to be used for open traverse



- Mtd of deflection angle is also used for open traverse
- Deflection angle is defined as an angle which a line makes with the prolongation of previous line

Note: once the traversing is completed, check must be applied over the measured interior angle

- If sum of measured interior angles is not equal to theoretical sum $\{(2N-4) \times 90^\circ\}$, then angle misclosure is attained. [Measured value - True value]
- If this angle misclosure is in permissible range then the measured angles are adjusted. & if it is large and out of permissible range, whole field work is to be repeated.
- The permissible angle misclosure depends upon the accuracy required and is given by

$$E = K\sqrt{N}$$

N = no of sides / angles of closed traverse

K = It is a constant which depends upon least count of theodolite

- The angle misclosure is generally distributed equally amongst all the angles (average correction mtd), but some other methods are also available

(A) ARBITRARY MTD

(B) ROUNDING OFF MTD

Corrections

for STATION	MEASURED ANGLES	C ₁	C ₂	C ₃	Corrected angle
A	101° 14' 20"	- 12"	- 20"	- 20"	101° 14'
B	230° 54' 20"	- 24"	- 20"	0	230° 54' 20"
C	16° 42' 00"	- 36"	- 40"	- 20"	16° 41' 40"
D	81° 35' 20"	- 48"	- 40"	0	81° 35' 20"
E	101° 35' 00"	- 60"	- 60"	- 20"	101° 34' 40"

$$\Sigma 540' 1''$$

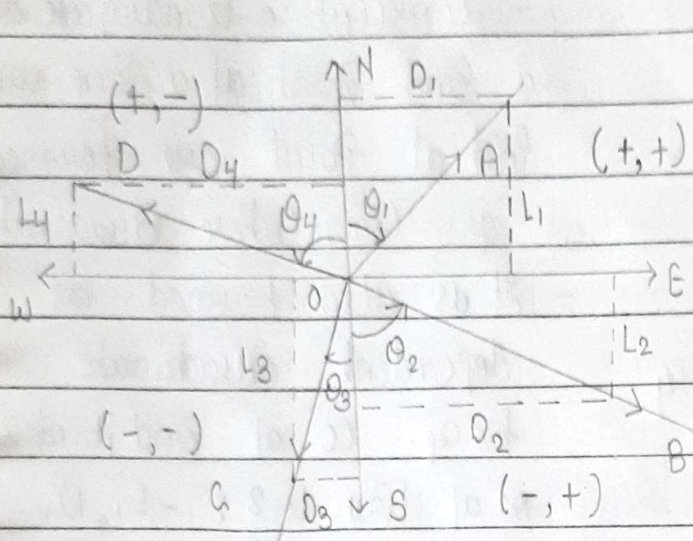
$$\text{error} = 540' 1'' - 540' = 1''$$

$$\text{avg correction} = 1''/5 = 20''$$

Spiral

By Jaspal Sir

LATITUDES & DEPARTURE



- The latitude^(L) of the line is the orthographic projection of the line on N-S axis representing the meridian

- The latitude is considered to be (+)ve if it is on north side and is termed as northing and is

considered to be (-) if it is on south side & is termed as SOUTHING

- The departure (D) of the line is the orthographic projection of the line on the axis perpendicular to the meridian i.e. E-W axis

- The departure is considered to be (+)ve if it is on East side and is termed as EASTING and is considered to be (-)ve if it is on West side and termed as WESTING

- Hence Latitude (L) = $l \cos \theta$

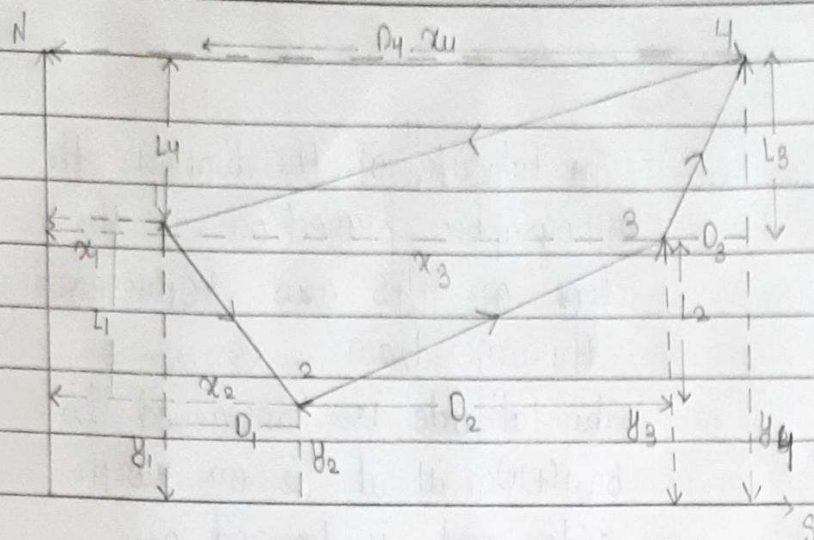
Departure (D) = $l \sin \theta$

θ : It is quadrantal bearing i.e. angle measured from N-S axis.

Note: If latitude and departure are computed with the help of quadrantal bearings respective signs are to be applied over it

- But if it is computed with the wcb, the result obtained would include the nature on its own.

CONSECUTIVE AND INDEPENDENT QUADRANTS



- Consecutive Quadrinate of a End Point of a line wrt Initial Point are termed as Consecutive Quadrinate
- It is also termed as dependent Quadrinate for eg. CC of End Point 2
 at line 1-2 : $-L_1, D_1$
 for Point 3 at line 2-3 : L_2, D_2

Independent Quadrinate : These are the Quadrinate of any Point measured w.r.t to Common origin

for eg IC of Point 1 : x_1, y_1

“ “ “ “ 2 : x_2, y_2

“ “ “ “ 3 : x_3, y_3

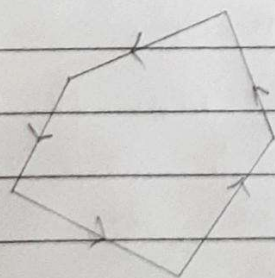
Note : $x_2 = x_1 + D_1$ $y_2 = y_1 - L_1$

$x_3 = x_1 + D_1 + D_2$ $y_3 = y_1 - L_1 + L_2$

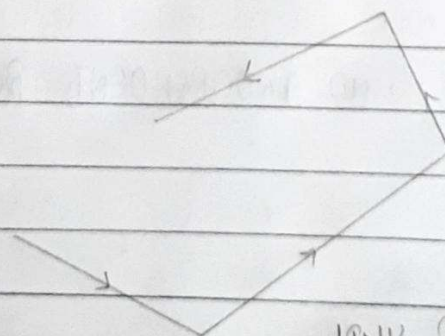
Hence Latitude of any Point = Latitude of start Point + sum of all latitude upto that Point

Departure of any Point = Departure of start Point + sum of all the departure upto that Point.

CHECK TO BE APPLIED IN CLOSED TRAVERSE



LOOP TRAVERSE



LINK TRAVERSE

(I) LOOP TRAVERSE:

- In closed loop traverse

(a) The algebraic sum of all the latitudes should be Equal to Equal

$$\sum L = L_1 + L_2 + L_3 + L_4 + \dots + L_n = 0$$

- It signifies that the arithmetic sum of Northing is Equal to Arithmetic sum of Southing

(b) The algebraic sum of all the departure must be Equal to zero

$$\sum D = D_1 + D_2 + \dots + D_n = 0$$

- It signifies that the arithmetic sum of Easting is Equal to Westing

(II) LINK TRAVERSE

In link traverse

(a) The algebraic sum of all latitudes must be Equal to difference of latitude of final and initial point

$$\sum L = L_1 + L_2 + L_3 + \dots + L_n = L_n - L_1$$

(b) The algebraic sum of all departure must be Equal to difference of departure of final and initial point

$$\sum D = D_1 + D_2 + \dots + D_n = D_n - D_1$$

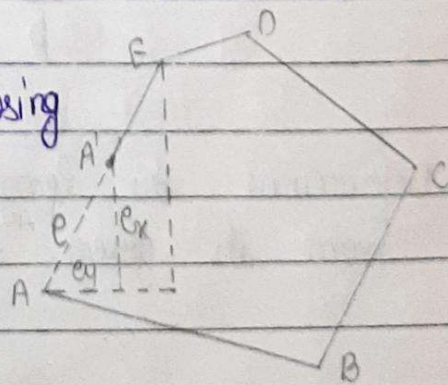
- If these ^{above} conditions are not being fulfilled in a closed traverse, it signifies the traverse has "Error of closure"

(c) LOOP TRAVERSE: If x & y Component of closing error are to be Computed, it is given

$$e_x = \sum D$$

$$e_y = \sum L$$

$$\text{Hence total closing error } e = \sqrt{e_x^2 + e_y^2}$$



Date.....

- Direction of closing error AA' , $\tan \theta = \frac{e_x}{e_y}$

The signs of e_x & e_y will define the quadrant of closing error.

- The correction to be applied would be just opposite to the error

$$C_x = -e_x$$

$$C_y = -e_y$$

- Hence closing error and correction will be in diagonally opposite quadrants

(b) LINK TRAVERSE

- The x & y component of closing error are given by

$$e_x = x' - x$$

$$e_y = y' - y$$

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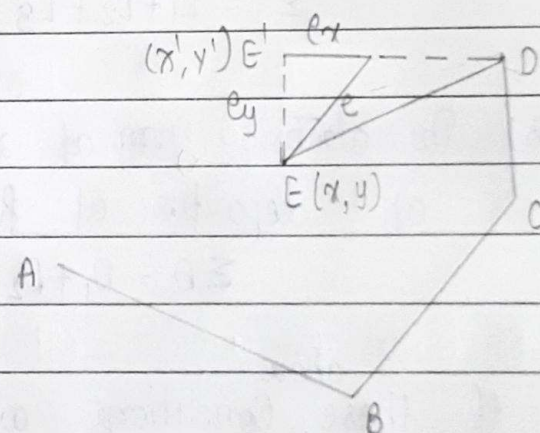
Here x' & y' are Computed Co-ordinates and x, y are Corresponding Known Co-ordinate.

Note: Relative Error of closure

It is defined as the ratio of error of closure to the Perimeter of traverse

- It is also termed as Relative accuracy or degree of accuracy

$$e_r = \frac{e}{p}$$



Q. = Determine the length of the line EA and its bearing from its given data

LINE	LENGTH (m)	BEARING	$L = L \cos \theta$	$D = L \sin \theta$
AB	194.1	$85^\circ 30'$	15.22	193.5
BC	201.2	150	194.34	52.07
CD	165.4	$285^\circ 30'$	44.2	-159.39
DE	172.6	$195^\circ 30'$	-166.32	-46.13
EA	—	—	α	y
			$\Sigma L = 87.45 + \alpha$	$\Sigma y = 40.06 + y$

For no error of closure $\Sigma L = 0$

$$\Sigma D = 0$$

$$87.45 + \alpha = 0$$

$$40.06 + y = 0$$

$$\alpha = -87.45 \text{ m}$$

$$y = -40.06 \text{ m}$$

$$L \cos \theta = -87.45$$

$$L \sin \theta = -40.06$$

$$L = \sqrt{(87.45)^2 + (40.06)^2}$$

$$L = 96.2 \text{ m}$$

$$\tan \theta = \frac{-40.06}{-87.45}$$

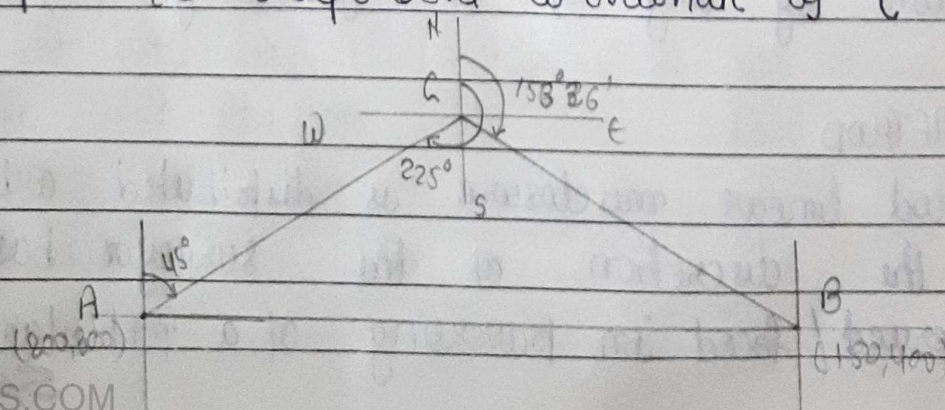
$$\theta = 204.61^\circ$$

$$\theta = 204.61^\circ$$

- Q) The Bearing of two inaccessible station taken from station C were 225° & $153^\circ 26'$ respectively. The Co-ordinate of A & B were as under

STATION	EASTING	NORTHING
A	300	200
B	400	150

Compute the Independent Co-ordinate of C



Date.....

$$\angle C = 225^\circ - 153^\circ 26' = 71^\circ 34'$$

$$\angle BAC = 225^\circ - 180^\circ = 45^\circ$$

$$\tan \theta = \frac{400 - 300}{200 - 150} = 2$$

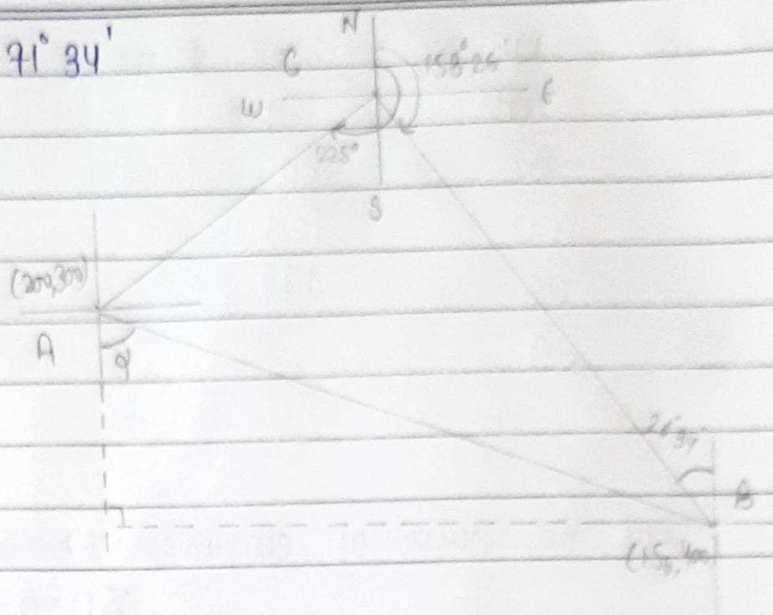
$$\theta = 63^\circ 26'$$

$$\angle A = 180^\circ - 45^\circ - 63^\circ 26'$$

$$\angle A = 71^\circ 34'$$

$$L_{AB} = \sqrt{(400 - 300)^2 + (200 - 150)^2}$$

$$L_{AB} = 111.8 \text{ m}$$



using sine rule $\frac{AB}{\sin C} = \frac{BC}{\sin A}$

$$BC = \frac{AB \sin A}{\sin C} = \frac{111.8 \sin(71^\circ 34')}{\sin(71^\circ 34')}$$

$$BC = 111.8 \text{ m}$$

$$\text{Northing of Point C} = 150 + 111.8 \cos 26^\circ 34' = 250 \text{ m}$$

$$\text{Easting of Point C} = 400 - 111.8 \sin 26^\circ 34' = 350 \text{ m}$$

ADJUSTMENT OF TRAVERSE

- If the error of closure is within the permissible limit then the traverse should be adjusted.
- The purpose of adjustment of the traverse is to reduce error of closure upto zero.
- It can be done by any of the following method.

(1) ARBITRARY METHOD

- In this method linear misclosure is distributed arbitrarily according to the discretion of the surveyor based on difficulty observed / faced in traversing of a particular line.

(8) BOWDITCH'S RULE/METHOD

- This method is used when linear and angular measurement are observed with same precision and accuracy.
- This method is also termed as "Compass Rule"
- This rule is based on assumption that the errors induced in the traverse are accidental in nature in which errors in linear measurement $\propto \sqrt{L}$ and errors in Angular $\propto 1/\sqrt{L}$

- In this method error is distributed on the basis of the length of traverse line

$$\text{Error in latitude of the line} = \frac{\text{Total error in latitude } (e_y) \times \text{length of the line}}{\text{Perimeter of traverse}}$$

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$$\text{Correction in latitude of any line } (C_{Li}) = \frac{\text{Total Correction in latitude } (-e_y) \times \text{length of the line } (l_i)}{\text{Perimeter of traverse } (\sum l_i)}$$

$$C_{Li} = -e_y \times \frac{l_i}{\sum l_i}$$

- Similarly Correction in departure of any line (C_{Di}) is

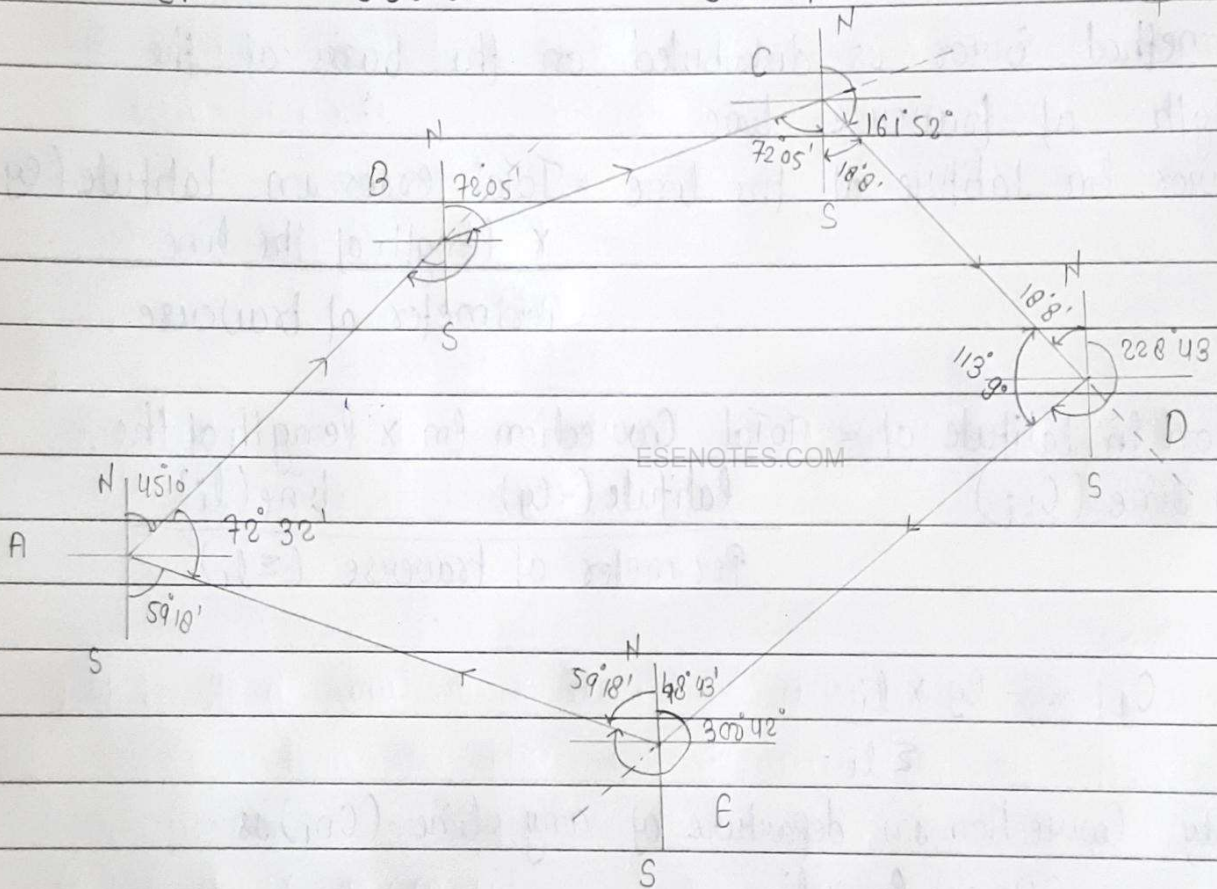
$$C_{Di} = -e_x \times \frac{l_i}{\sum l_i}$$

Note: When a traverse is adjusted as per this rule both length and bearing of the line gets changed

Note: GRAPHICAL MTD: This method is also based upon Bowditch's rule, with the only diff. that correction is applied graphically.

Q) Calculate latitudes, departure and closing errors for the following traverse and adjust using Bowditch rule

LINE	LENGTH	WCB
AB	89.31	$45^{\circ} 10'$
BC	219.76	$72^{\circ} 05'$
CD	151.18	$161^{\circ} 52'$
DE	159.10	$228^{\circ} 43'$
EA	232.26	$300^{\circ} 42'$



$$\angle A = 72^{\circ} 32'$$

$$\angle B = 45^{\circ} 10' + 180^{\circ} - 72^{\circ} 05' = 153^{\circ} 5'$$

$$\angle C = 72^{\circ} 05' + 180^{\circ} - 161^{\circ} 52' = 90^{\circ} 13'$$

$$\angle D = 161^{\circ} 52' + 180^{\circ} - 228^{\circ} 43' = 113^{\circ} 9'$$

$$\angle E = (228^{\circ} 43' - 180^{\circ}) + 300^{\circ} 42' - 360^{\circ} \text{ or } 59^{\circ} 18' + 48^{\circ} 43' = 108^{\circ} 1'$$

Summation of all interior Angle = $\angle A + \angle B + \angle C + \angle D + \angle E = 540^{\circ}$

$$\text{Theoretical sum} = (2n-4) \times 90 = (2 \times 5 - 4) 90 = 540^{\circ}$$

$\Rightarrow$ Hence there is no angle misclosure

LINE	LATITUDE	DEPARTURE
AB	62.97	63.33
BC	67.60	209.10
CD	-143.87	47.05
DE	119.88 -104.97	-119.56
EA	118.58 118.58	-199.70

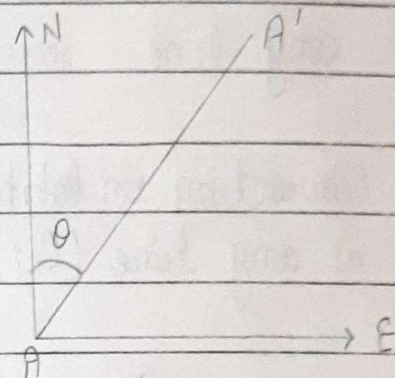
$$\Sigma L = 0.51 \quad \Sigma D = 0.22$$

$$e_x = \Sigma D = 0.22, \quad e_y = \Sigma L = 0.51$$

$$e = \sqrt{e_x^2 + e_y^2} = \sqrt{0.22^2 + 0.51^2} = 0.55$$

$$\tan \theta = \frac{e_x}{e_y} = \frac{0.22}{0.51} = 0.431$$

$$\theta = 23^\circ 20'$$



- using Bowditch rule

$$\Sigma L_i = 851.61 \text{ m}$$

LINE	Latitude Correction	Departure Correction	Corrected Latitude	Corrected Departure
AB	$0.51 \times \frac{62.97}{851.61} = -0.053$	$-0.22 \times \frac{63.33}{851.67} = -0.023$	62.92	63.31
BC	-0.132	-0.057	67.49	209.04
CD	-0.09	-0.04	-143.76	47.01
DE	-0.095	-0.04	-105.06	-119.60
EA	-0.14	-0.06	118.44	-199.76

(d) TRANSIT RULE

- This method is used to balance a traverse in which angular measurement are more precise than the linear measurement. In this method error is distributed on the basis of latitude / departure of the traverse line.

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- In the transit rule, the angles are changed less but length are changed comparatively more.
- Note: However in bowditch rule in comparison to transit rule, change in length is less, but change in angle is more.

$$\text{Error in latitude of any line} = \frac{\text{Total Error (e}_y\text{) in latitude}}{\text{Arithmetic sum of latitude}} \times \text{Numerical value of latitude}$$

$$\text{Correction in latitude of any line (C}_i\text{)} = \frac{\text{Total Correction in latitude (-e}_y\text{)}}{\sum |L_i|} \times \text{Numerical value of latitude (|L}_i\text{|)}$$

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$$C_i = \frac{-e_y |L_i|}{\sum |L_i|}$$

Similarly Correction in departure of any line (C_{o_i})

$$C_{o_i} = \frac{-e_x |D_i|}{\sum |D_i|}$$

Note: If a line has zero latitude / departure, there may be no correction to the latitude / departure of that line

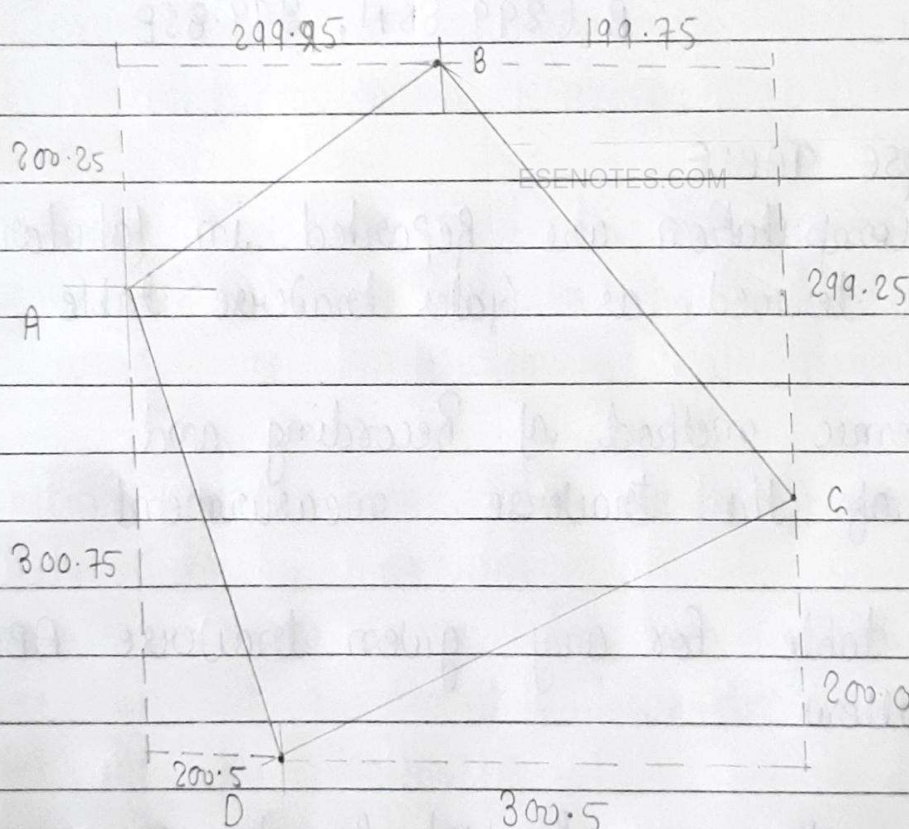
(E) AXIS METHOD

- In this method the corrections are applied only to the length
- This method is used when measurement of the angle is done with very high precision

Q) following table gives data of Consecutive Co-ordinate in respect to a closed traverse ABCDA

STATION	N	S	E	W
A	300.75			200.5
B	200.25		299.25	
C		299.0	199.75	
D		200.0		300.5

- Calculate (i) Magnitude & direction of closing Error
 (ii) Corrected Consecutive Co-ordinate of stations
 (iii) Independent Co-ordinate of the station B, if those of A are (100, 100), Use transit rule.



$$\sum L = 2 = e_y$$

$$\sum D = -2 = e_x$$

$$e = \sqrt{e_x^2 + e_y^2} = \sqrt{2^2 + (-2)^2} = 2\sqrt{2} = 2.828$$

$$\tan \theta = \frac{e_x}{e_y} = \frac{-2}{2} = -1 \Rightarrow \theta = 315^\circ$$

(ii) Correction $C_{LAB} = \frac{-2 \times 200 \cdot 25}{(200 \cdot 25 + 299 + 200 + 300 \cdot 75)}$

$$C_{LAB} = -0.4$$

$$C_{DAB} = \frac{-(-2) \times 299 \cdot 25}{1000} = +0.6$$

~~LAB~~ Corrected $L_{AB} = 200 \cdot 25 - 0.4 = 199.85$

Corrected $D_{AB} = 299 \cdot 25 + 0.6 = 299.85$

Consecutive Coordinate of B (199.85N, 299.85E)

(iii) Independent Coordinate of B (100+199.85N, 100+299.85E)
B (299.85N, 399.85E)

GALES TRAVERSE TABLE

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When traverse computation are reported in tabular form then it is termed as Gales traverse table

- It provide systematic method of Recording and Computation of the traverse measurement
- Gales traverse table for any given traverse ABCDEA is as follow

(i) The following steps are involved in traversing

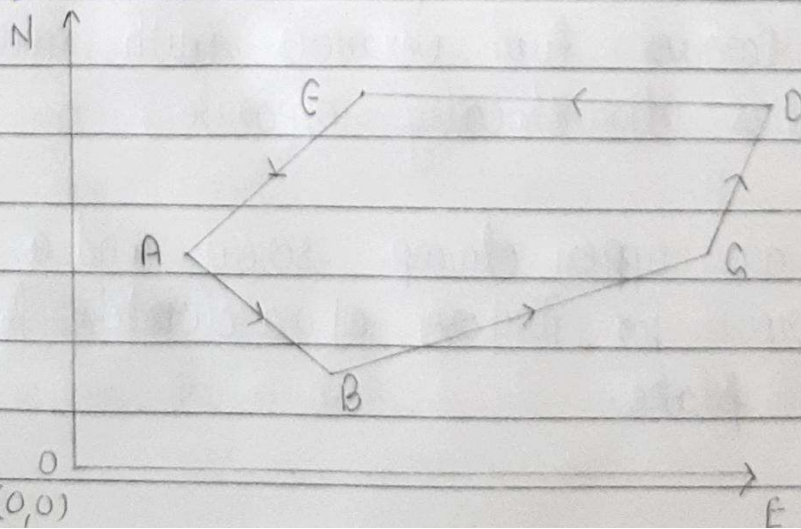
- In case of theodolite traverse the included angles are computed and adjusted to satisfy the geometrical computation.

i.e. sum of Interior Angle = $(2N-4) \times 90$

or sum of Exterior Angle = $(2N+4) \times 90$

- In Case of Compass traverse the observed bearing are adjusted for local attraction
- With the help of observed bearing of the traverse line, Compute the bearing of other line if missing and Reduced it to Quadrantal system.
- With the help of length and Computed Reduced bearing of the line the Consecutive Coordinates i.e latitude and departure are Computed
- A check is Performed to find out wheather the Error of closure Exist in traverse or not i.e algebraic sum of latitude and departure must Come out to be zero
- If Error of closure is find, Correction is to applied over it by a Particular method depending upon accuracy in measurement of direction and length.
- The Independent Coordinate are then worked out from Consecutive Co-ordinate

Note: The origin is selected such that the Entire traverse line in NE Quadrant, it is done for the ease of plotting of traverse



ASSEMBLY OF THEODOLITE

The Entire theodolite Can be divided Into three Assemblies.

(i) LEVELLING HEAD

It Consist of two Parallel plates separated by 3 levelling screws. The upper Parallel plates of levelling head is known as Tribach. and the lower one is known as Trivet / Foot plate.

- The levelling screw work in Threaded holes in the Tribach Arms and their lower ends fits in Recesses in the Trivet.
- The trivet is a Casting having a large hole in the Centre with the Threads.
- The trivet Can be screwed on to metal Casting screw at the tripod when mounting the theodolite.
- The trivet has a hook at the lower end of the inner spindle from where the Plumb bob Can be suspended.

(ii) The upper plate or Vernier plate.

- The upper plate or Vernier plate is attached to the inner axis and carries two Verniers with magnifiers at two Extremities diametrically opposite.
- It carries an upper clamp screw and a corresponding tangent screw for Purpose of accurately fixing it to lower plate.

- on clamping the upper and unclamping the lower clamp, the instrument can rotate on its outer axis without any relative motion between the two plates
- If however the lower clamp is clamped and upper clamp is unclamped, the upper plate and the instrument can rotate on the inner axis with a relative motion between a vernier and a scale. For using any tangent screw, its corresponding clamp screw must be tightened.

(b) LOWER PLATE / MAIN SCALE PLATE

- It is mounted on a hollow spindle, also called outer spindle
- The lower plate is graduated in degree from $0-360^\circ$ in clockwise direction with least count $20'$
- The lower plate is provided with lower clamp screws, which if tightened, the outer spindle is fixed to tribrach and hence lower to plate is fixed in position

(c) PLATE LEVEL / TUBE LEVEL

A Tube level is mounted on the upper plate

- Bubble is centered by rotating levelling screw
- These plate levels are provided to make vertical axis of the instrument truly vertical.
- Plate level kept parallel to turnion axis

(3) ALIDADE ASSEMBLY

It consists of following

(A) TELESCOPE : It is Mounted on horizontal spindle called as H-axis / Turnion axis

- The Telescope Can be rotated in vertical plane about this horizontal axis for Purpose of taking / sighting the object.

(B) VERNIER FRAME / INDEX FRAME / T-FRAME : It consist of a vertical leg called clipping arm and a horizontal bar called Index Arm.

- Vernier C and D are fitted at the two ends of the Index frame.
- It is Provided in front of vertical circle which has main scale attached with it that rotates along with the telescope.
- The vertical circle and the telescope can be clamped at any desired vertical angle by means of vertical clamp screw and for small movement / Rotation vertical tangent screw can be used.

(C) ALTITUDE BUBBLE : It is attached to the Index arm hence to make the Index arm truly horizontal during adjustment [temporary], It is used.

- Its Sensitivity is more than Plate bubble, due to the fact that the Entire Instrument need not be levelled before taking Every Reading.

DEFINITION AND TERMS

(A) HORIZONTAL AXES / TURNION AND TRAVERSE AXIS

- It is the axis about which telescope can rotate in a Horizontal plane

(b) VERTICAL AXIS / AZIMUTH AXIS: It is the axis about which telescope can be rotated in vertical axis.

(c) LINE OF SIGHT: It is the imaginary line passing through the intersection of the cross hairs and of the diaphragm and the optical centre of the objective.

- when line of sight comes in horizontal plane it is called line of collimation.

(d) AXIS OF LEVEL TUBE: It is the line tangential to the longitudinal curve of the level tube at its centre.

- Axis of plate level is horizontal when bubble is at centre.

- It is parallel to horizontal axis and perpendicular to vertical axis.

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Note: line of sight, horizontal axis, vertical axis should be perpendicular to each other.

(e) FACE RIGHT AND FACE LEFT: when the vernicle circle of theodolite is on right end side of the observer, the position is called face right and the observation made is called face right observation.

- However when vernicle circle of theodolite is on left end side of the observer, the position is called face left and the observation made is called face left observation.

(f) TRANSIT / PLUNGING / REVERSING: It is the process in which telescope is revolved around about horizontal axis in vertical plane by an

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Angle of 180° , thus making it point in just opposite direction.

(G) SWING : It is the operation of revolving the telescope in the horizontal plane about vertical axis.

- If swing is done in clockwise direction it is termed as right swing and if it is done in Anticlock-wise direction, it is termed as left swing.

(H) CHANGING FACE : It is the ~~observation~~ operation of bringing the telescope from face left condition to face right condition and vice-versa.

- It is done by plunging and by swinging it by 180° . By changing the face following errors can be eliminated-

- Error due to ^{line of} collimation not being perpendicular to horizontal axis.
- Error due to Horizontal axis not being perpendicular to ~~horizontal~~ vertical axis.
- Error due to line of collimation not being parallel to axes of the level.

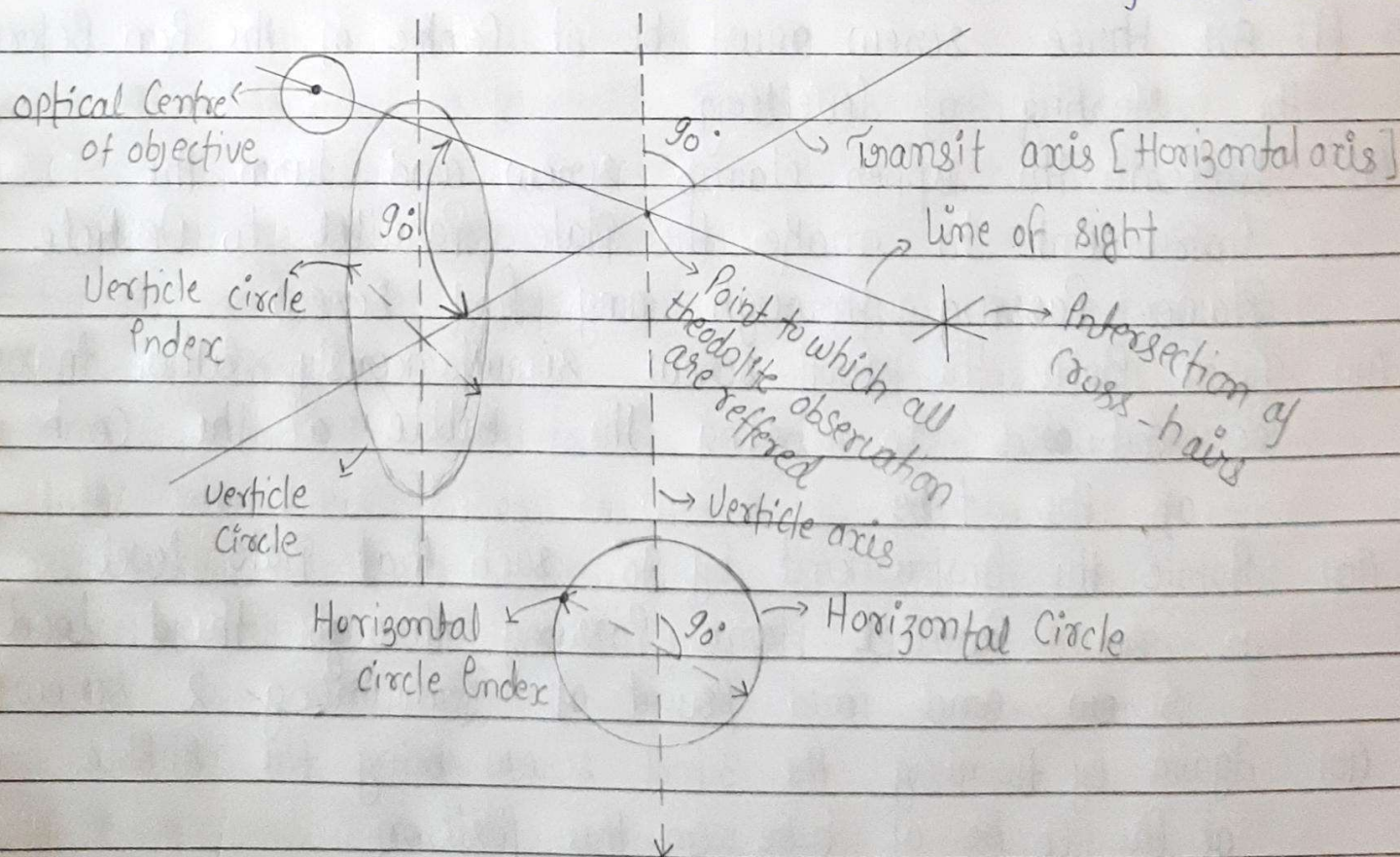
Note : If horizontal and vertical angle are measured twice once with telescope normal and once with telescope inverted is termed as Double sighting.

FUNDAMENTAL LINE AND DESIRED RELATION

4 lines are fundamental line

- | | |
|-------------------------------|----------------------------|
| (i) The vertical axis | (v) Axis of altitude level |
| (ii) The horizontal axis | (vi) Axis of spirit level. |
| (iii) The line of collimation | |
| (iv) Axis of plate level | |

- Horizontal circle must be perpendicular to vertical axis.
- Vertical circle must be perpendicular to Horizontal Axis.
- Vertical Axis must pass through the Centre of Horizontal Circle.
- Horizontal axis must pass through the Centre of Vertical circle.
- Axes of plate level ~~must~~ must be perpendicular to vertical axis.
- Horizontal axis must be perpendicular to vertical axis.
- Line of sight must be perpendicular to Horizontal axis.
- For horizontal position of telescope and for altitude bubble at the Centre the Reading on the Vertical Circle must be zero.
- Line of sight, Vertical axis and horizontal axis must meet at Common Point termed as Centre of Instrument.



TEMPORARY ADJUSTMENT of THEODOLITE

(1) **SETTING UP**: Station the tripod. legs should be placed that each make an Angle 60° with Ground Surface [Horizontal]

- By moving the leg radially, the plumb bob is shifted in the direction of the leg, while by moving leg circumferentially or side ways Considerable change in Inclination is effected without disturbing the plumb bob.

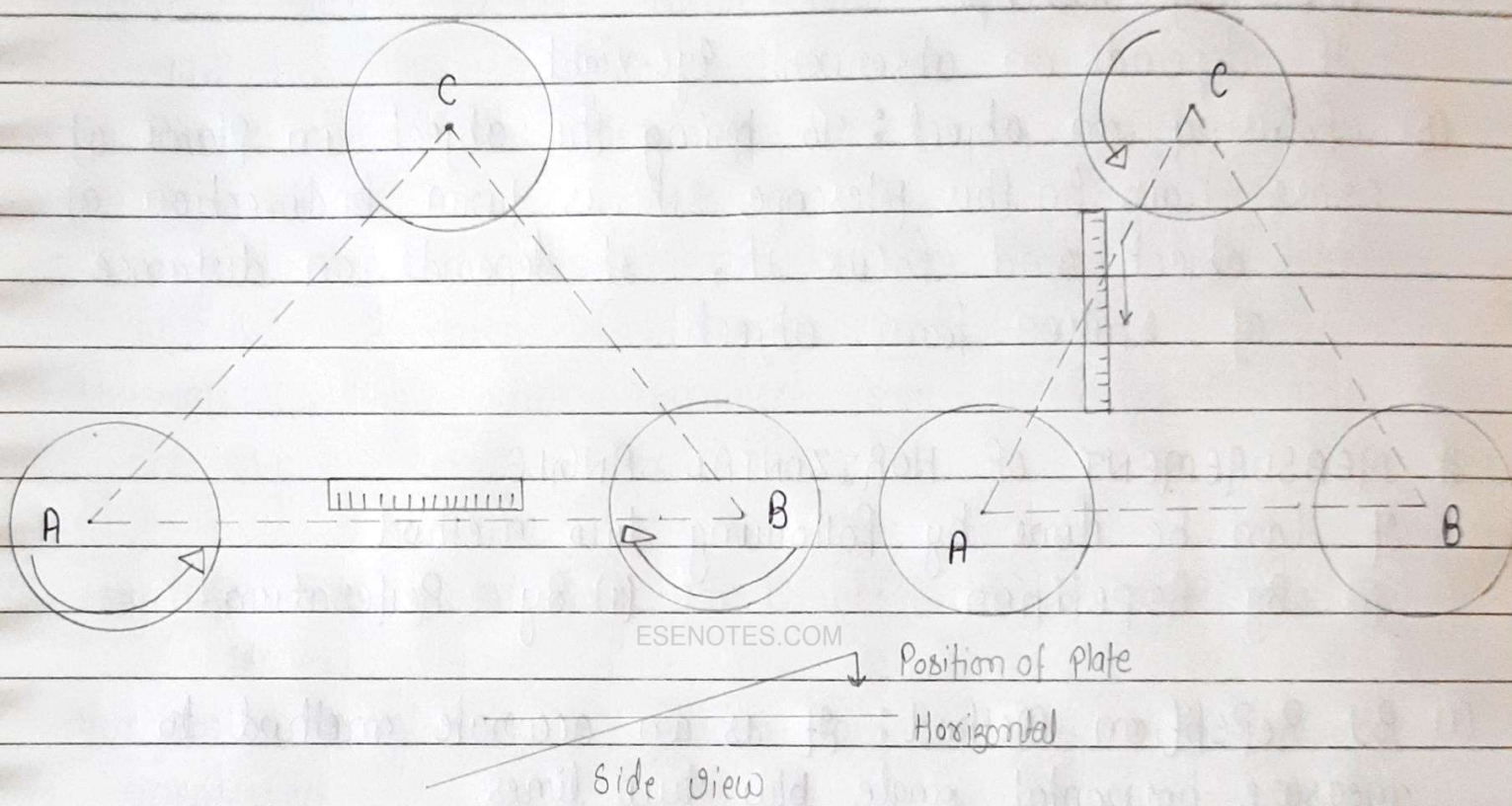
(b) **LEVELLING UP**: It is done with the help of Levelling screw

- It is done to make vertical axis truly vertical and horizontal plate truly horizontal.
- Basic Principle involved here is to make the axis of plate level horizontal.

It is done as follow

- (i) All three screw must be at Centre of the Run Before the starting of levelling
- (ii) Loosen the upper clamp screw and turn the instrument to make the plate Parallel to Vertical Plane Passing through any foot screw
- (iii) Turn Both the foot screw simultaneously either inward or outward to bring the bubble at the Centre of the tube.
- (iv) Rotate the instrument by 90° such that Plate level is now Parallel Plane Passing through third foot screw and mid point of Remaining 2 screw
- (v) Again by turning the 3 foot screw bring the bubble at the Centre of tube in this position

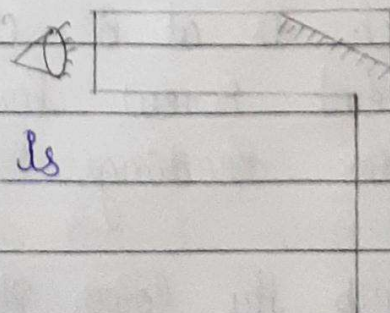
- (6) Repeat step (4) and (5) upto an extent bubble is at Centre of tube in both the position
- (7) Turn the instrument by 180° and check wheather the bubble is at Centre of tube or not if not Permanent adjustment will be Required.



(2) CENTERING: It is done to place the vertical axis exactly over the station mark. It can be done by following two methods

- By Circumferential movement of tripod
- Radial movement of tripod
- Plumb bob does not shift But level will differ
- Plumb bob also shift

OPTICAL PLUMMET: It is used to measure that theodolite is exactly at center.



FOCUSING

- (a) Focusing on Eye Piece: Cross hair of diaphragm should be seen clearly and distinct
To see clear eye piece turn the telescope indirection of sky and put a white paper in front of telescope
- It depends on observer eye-sight
- (b) Focusing on object: To bring the object in plane of cross hair in this telescope it is turn indirection of object and focus it, it depends on distance of station from object

MEASUREMENT OF HORIZONTAL ANGLE

It can be done by following two method

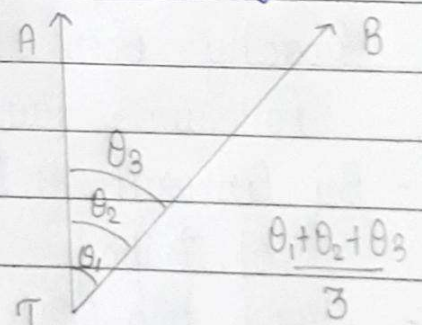
- (i) By Repetition (ii) By Reiteration

(i) By Repetition Method: It is an accurate method to measure horizontal angle b/w two lines

- In this same angle is measured multiple times
- The Arithmetic mean of observation gives true value of an angle

Step:- Set the instrument at T, clamp the upper plate. use upper plate tangent screw for exact bisection.

Rotate the telescope in horizontal plane [because lower screw is free] sight the staff which is at B. clamp the lower screw and use tangent screw for exact bisection and measures vernier readings



- unclamp the lower plate and turn telescope to sight B again Tighten the lower clamp use lower plate tangent screw for

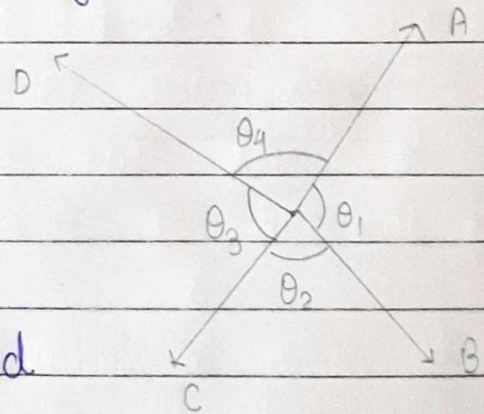
Exact bisection of the B. There Vernier Reading should be ~~same~~ as it was during previous.

- Release the upper clamp and turn the telescope to sight B. Tighten upper clamp bisect B exactly using upper telescope.
- Repeat same process again and final of Vernier A will be approximately twice θ .
- Divide the final Reading By no of Repetition to obtain AOB
- Change face of instrument to face Right the telescope will be Inverted Condition Repeat steps.
- Determine avg Value obtained from face left and face Right. This give Exact θ .

(2) By Reiteration method or Direct method or Method of series.

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Horizontal angle of a group having Vertex Point Can be measured



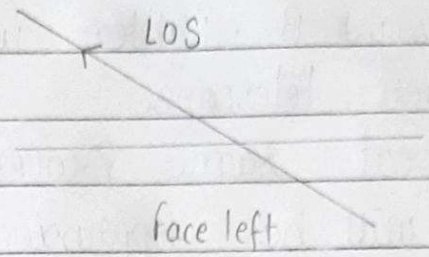
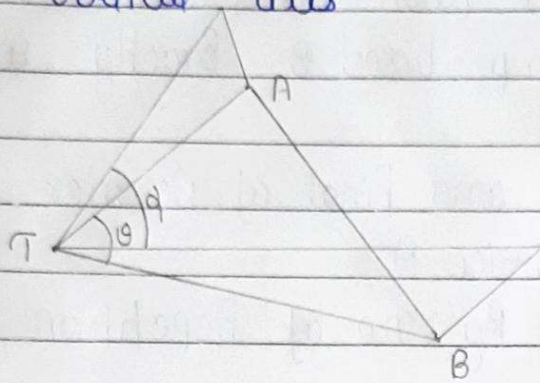
ERRORS IN THEODOLITE

Errors in theodolite can be induced due to following

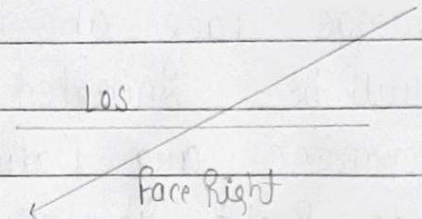
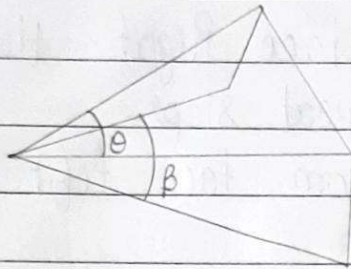
- (i) Instrumental Error
- (ii) Mistake
- (iii) Natural Error

(A) INSTRUMENTAL ERROR: It is caused due to Construction or manufacturing defects or due to wear and tear of various parts of instrument.

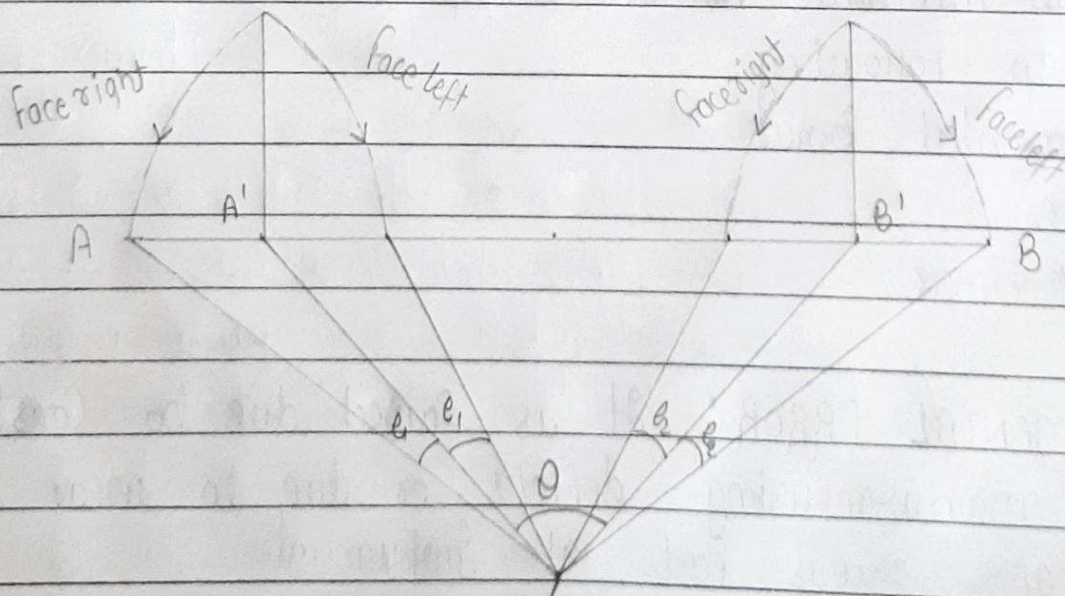
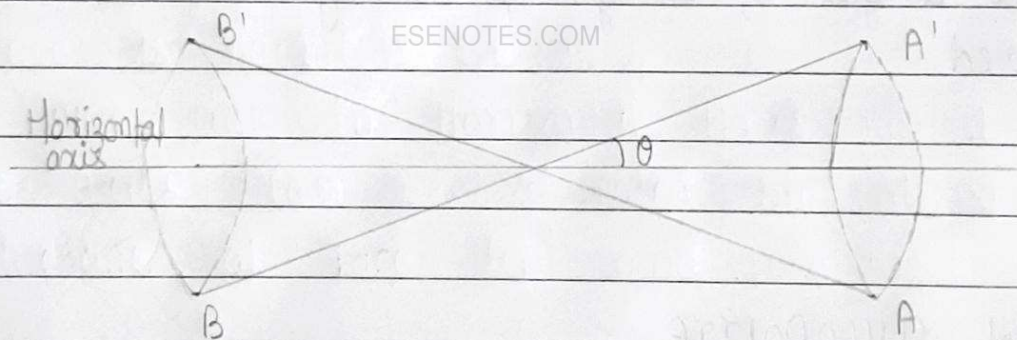
- Error due to plate bubble not perpendicular to the vertical axis



$$\theta = \frac{\alpha + \beta}{2}$$

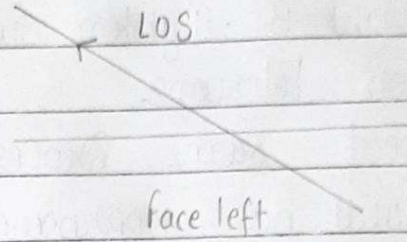
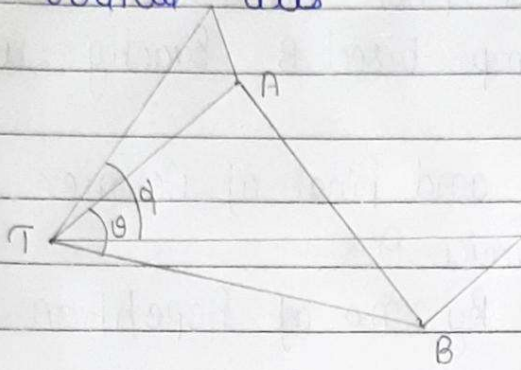


(b) Error due to line of sight not being perpendicular to horizontal axes

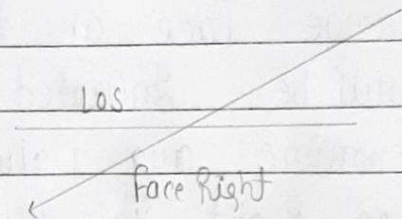
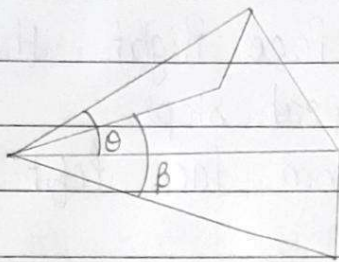


These errors include

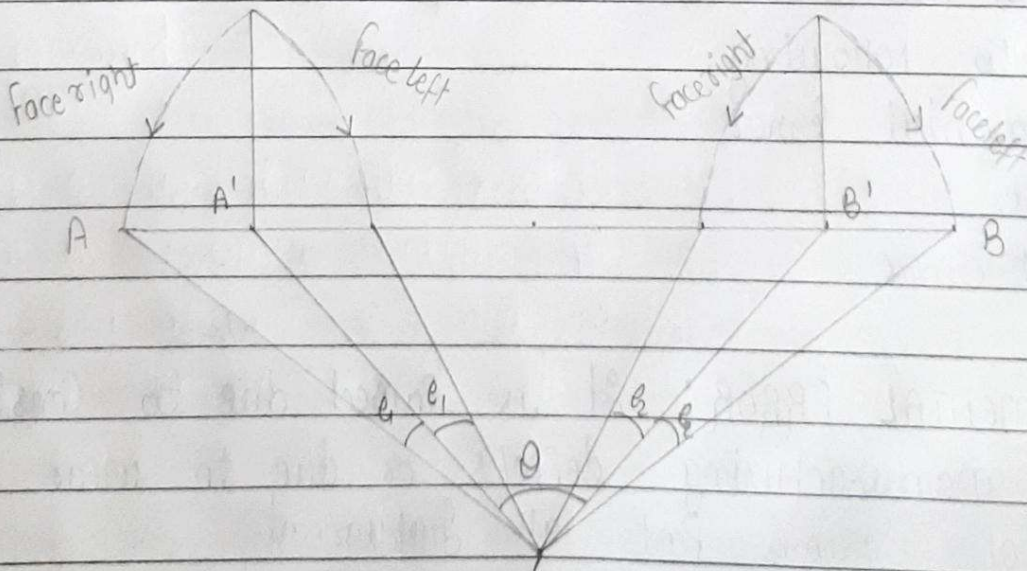
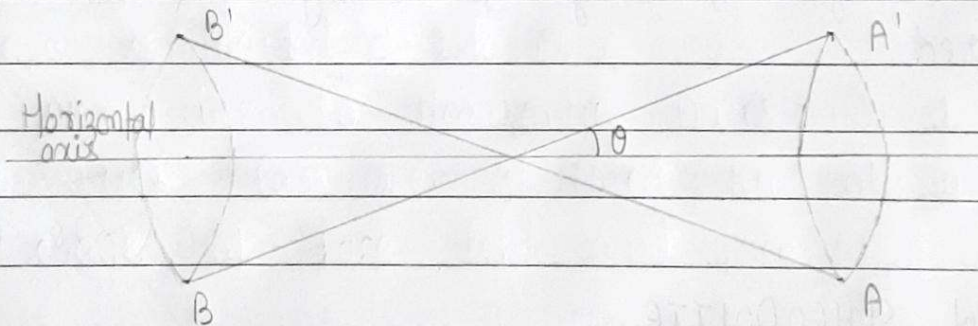
- Error due to plate bubble not perpendicular to the vertical axis



$$\theta = \frac{\alpha + \beta}{2}$$



- (b) Error due to line of sight not being perpendicular to horizontal axes



Angle measured b/w A and B in Face left Position

$$\theta_1 = \theta - (e_1 + e_2) \quad \text{--- (i)}$$

Angle measured b/w A and B in Face right Position

$$\theta_2 = \theta - e_2 - e_1 \quad \text{--- (ii)}$$

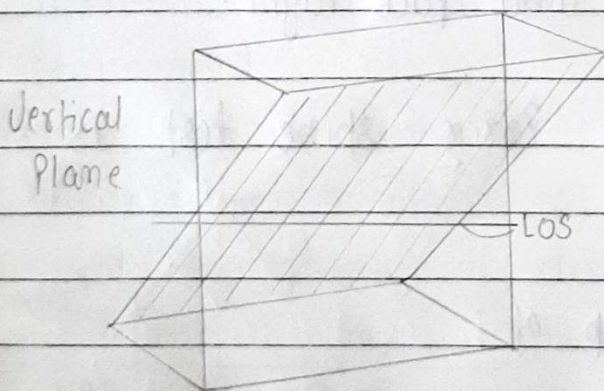
Taking avg. of Angle in Face left and right

$$\frac{\theta_1 + \theta_2}{2} = \frac{\theta - e_1 + e_2 + \theta - e_2 + e_1}{2} = \theta$$

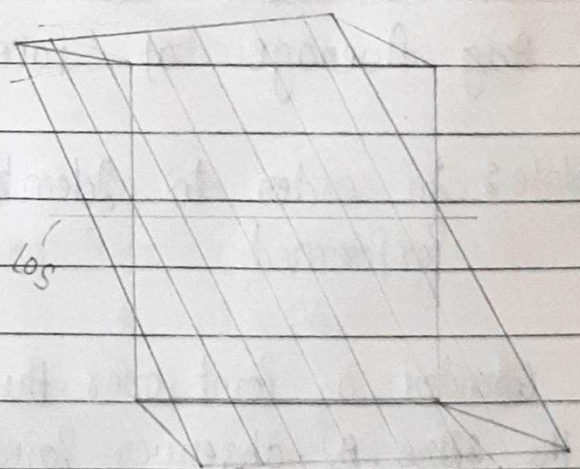
- In this case line of sight does not Revolve in Vertical plane and the telescope is lifted and lowered in the case it will generate a Cone of axes which will ~~can~~ coincide with the horizontal axes.

- The Error due to line of sight not being Perpendicular to horizontal axes can be Eliminated by taking an avg of face left and Face right measurement i.e by double sighting.

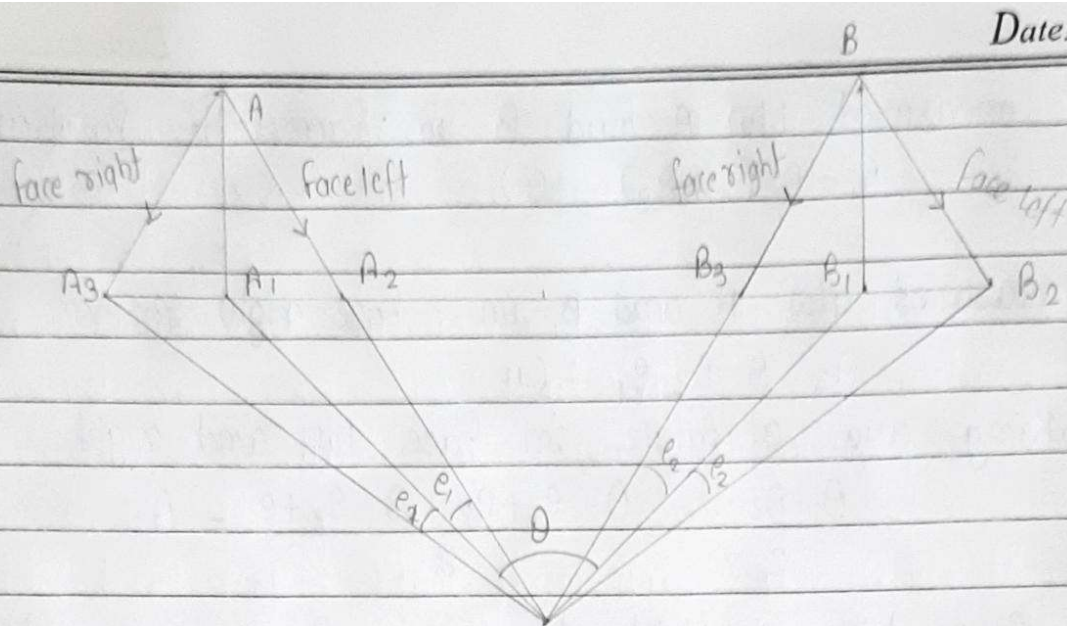
(c) Error due to Horizontal axes not being Perpendicular to Vertical axes



Inclined Position [for Face left Position]



Vertical plane



Angle measured A & B in face left position

$$\theta_1 = \theta - e_1 + e_2$$

Angle measured b/w A & B in face right & left position

$$\theta_2 = \theta + e_1 - e_2$$

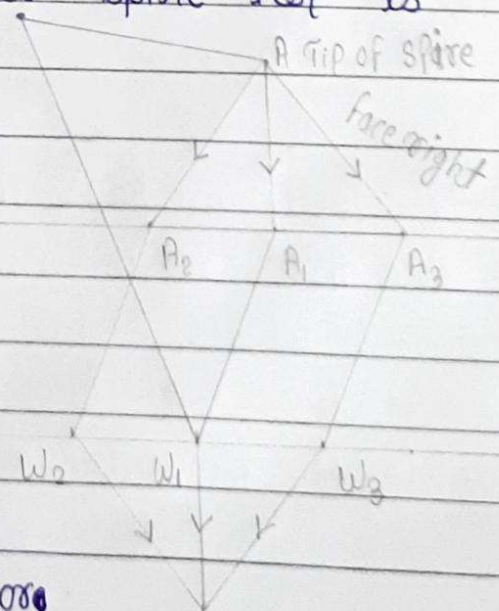
$$\text{Avg} = \frac{\theta_1 + \theta_2}{2} = \frac{\theta - e_1 + e_2 + \theta + e_1 - e_2}{2} = \theta$$

If the horizontal axis is not perpendicular to the vertical axis, the line of sight moves in the inclined plane when the telescope is raised or lowered.

Thus error due to horizontal axis not $\perp$ to the vertical axis can be eliminated by taking the average of face left and face right.

Note: In order to identify this error, a sphere test is performed.

Consider a point over the point tip of the sphere (A). observer point (A) from instrument station "O" and bring in the horizontal position it will not come exactly at point A, because of obstruction [wall] it will be W₂. Same process repeat for



Face Left and Face Right and take avg of w_2 and w_3 and which will be w_1

If the Projection of A in horizontal surface in both Condition [Face Left and Face Right] is the Point on the ground surface comes out to be same it means there is no error due to not Perpendicular Horizontal and Vertical axes to each other

VERTICAL INDEX ERROR

- Axes of the altitude level tube must be truly horizontal when the Bubble is in Center.
- The line of collimation be Parallel to the axis of the tube when Vertical circle read 0 but it is not Vertical. Vernier C&D would not Read zero when line of sight is Horizontal
- This error is Eliminated by taking average of Readings of Face Left and Face Right measurement.

ERROR DUE TO IMPERFECT GRADUATION ON CIRCLE

- Imperfect Graduation on Circle are either due to manufacturing defects or due to Continuous use of Instrument
- Error due to Imperfect Graduation on Circle is Eliminated either by method of Repetition or By taking the observation in different Parts of Circle.

PERSONAL ERROR

- (A) Error due to Inaccurate Centering
- (B) Error ee ee ee Levelling
- (C) Error ee ee ee slip of clamp screw
- (d) Error ee ee ~~Improper~~ Improper use of tangent screw
- (E) Error ee ee Refraction.

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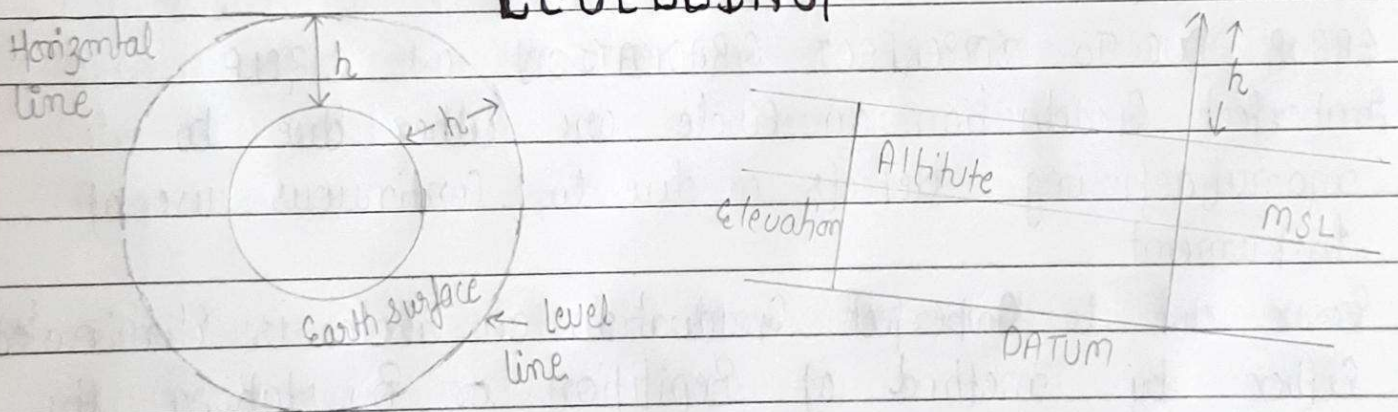
Note: Error due to Refraction and temperature may be Categorized Error due to natural forces.

Note: The Permanent adjustment of theodolite are so arranged in sequence that the next adjustment does not disturb the previous adjustment.

The order is given as -

- (A) Plate level test - Plate level axis Perpendicular to Vertical axis.
- (B) Cross-Hair ring test - Vertical cross hair Perpendicular to Horizontal axes
- (C) Collimation: Horizontal Perpendicular to Vertical
- (d) Spire: Horizontal ee ee ee
- (e) Bubble tube test: Line of sight Exactly horizontal
- (f) Vertical arc test: Vertical circle indicate '0' when the line of sight is Perpendicular to Vertical axis.

LEVELLING



Levelling is the process in which comparison of Heights of points on the surface of the earth is carried out.

- If all the Heights of different points are measured relative to a plane, the plane is termed as Datum Plane.

- In General the datum plane is taken to measure mean sea level (MSL), to standardise the measurement of Height Internationally.
- If the vertical distance of the point above or below the datum surface is noted it is termed as Elevation and if it noted w.r.t mean sea level it is termed as Altitude.

Note: If Datum is MSL then Elevation = Altitude

- Here mean sea level is obtained by averaging level of high and low tide at different on the sea for ~~the~~ a period of 19 years.
- A level line is one which is constant height above 'MSL'
- But Horizontal line is tangential to the level line at any given point
- whereas vertical line for any point on the earth surface passes through the centre and is represented by plumb-bob

BENCH MARK:

It is a fixed point of known elevation above the datum

These are further classified as follow

(a) Great Trigonometrical Survey (GTS)

The Great Trigonometrical Survey Bench Mark are those which are established by survey of India with very high precision or accuracy throughout the country

(b) Permanent Bench Mark: These Bench Mark are established by PWD or survey of India at an interval closer to that of GTS B.M and is used for local survey

(c) Temporary B.M. : These are Established temporarily as when required to continue to the Execution of the previous day or work.

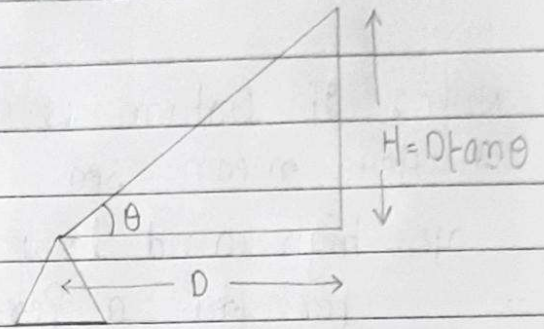
(d) Arbitrary B.M. : These are B.M whose Elevation is assumed to Arbitrary for small survey.

TYPES OF LEVELLING

levelling can be classified as

(1) DIRECT LEVELLING : Most Common form of levelling Also termed as spirit levelling.

- In this case levelling is done with the help of spirit and telescope.



(2) TRIGONOMETRIC LEVELLING : This type of levelling is used when the point under consideration is difficult to access Ex- Top of towers, mountain, etc.

- Here the diff. in Elevation is determined indirectly from the horizontal distance and vertical angle of concern point.

(3) BAROMETRIC LEVELLING : It is also indirect type of levelling in which the Elevation of various points are determined indirectly from the changes in the atmospheric pressure [ATM ↓ with ↑ height]

- Instead of using Barometer for measuring pressure Altimeters can also be used.

(d) HYPSOMETRIC LEVELLING : It is also a type of indirect levelling in which diff. of Height is measured with noting the diff. in boiling point of water [with H ↑ BP ↓]

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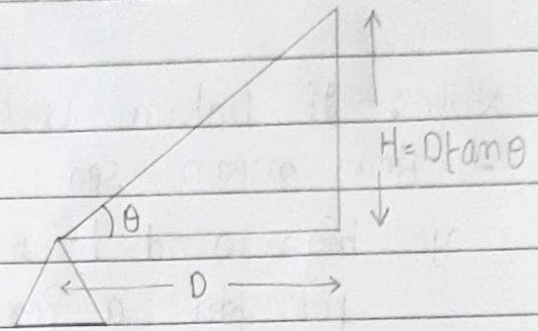
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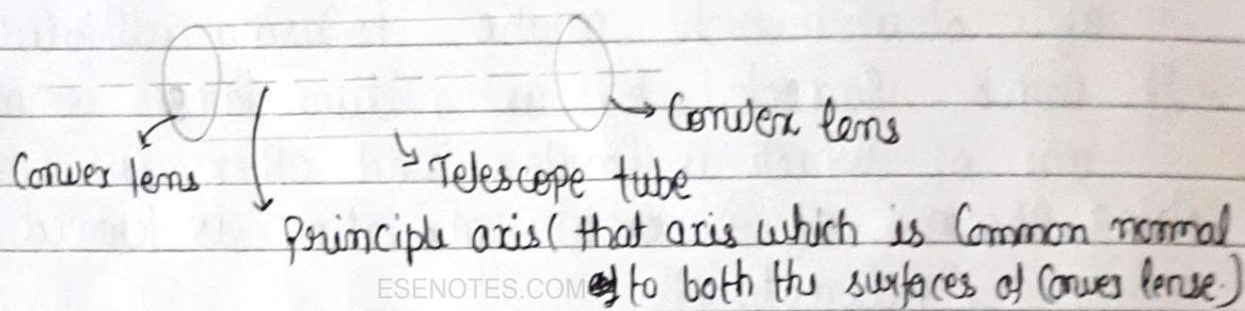
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Note: Here direct Levelling is classified as

- a) Simple levelling
- b) Differential levelling
- c) check
- d) Profile levelling
- e) C/S levelling
- f) Fly levelling
- g) Reciprocal levelling.

Levelling is done with the help of following Instrument

- (i) Telescope
- (ii) Bubble Tube



Optical Centre: Generally located near to Geometric Centre

The Rays which Passes through Center does not deviate

Optical axis: A line Passing through optical Centre and Centre of the Curvature of both the lenses

Telescope: It is an Instrument used for sighting the object

- Commonly used telescope in surveying is Kepler type telescope
- It Consist of Convex lenses on either side of the tube
- The Principal axis of the both the lenses must be in same line

Line of Sight: Line joining Intersection of the cross hair to optical Center of object.

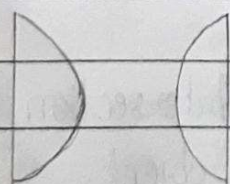
- Principal axis is the Common normal to both the surfaces of the lens
- The optical Center may be defined as the Point through which all the rays of light Passes without getting deviated.
- The line Passing through optical Centre and Centre of Curvature of the lens is Called optical axis.
- Lenses used in telescope may suffer following defects:-
- (A) CHROMATIC ABERRATION: This defect occurs when white light split into its seven Components
- The Image is therefore surrounded by Rainbow of Colour which make focusing difficult
- It Can be Corrected by using two lenses joint together one of which is Convex and other is Concave.

Note: Absence of chromatic aberration is termed as "Achromatism"

(b) SPHERICAL ABERRATION: This defect occurs due to spherical surface of lens.

- Due to spherical surface of lens the Rays of light Incident at the edge of lens are Reflected more than the rays near to the Center of lens
- This Effect Can be Corrected ~~used~~ using Combined Lenses.
- Absence of spherical aberration is termed as "Aplanation"

Note: Most Commonly used Eye Piece is termed As RAMSDEN Eyepiece. It is made up of two Identical Plane of Convex ~~to~~ lens with their Curved surface Pointed towards Each - other



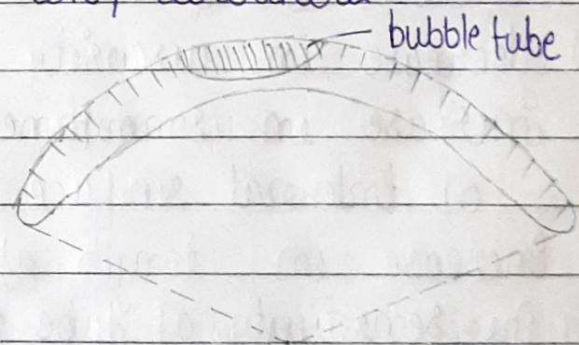
This Eye-Piece is free from spherical aberation but has Chromatic aberation.

(C) LEVEL TUBE / BUBBLE TUBE / SPIRIT LEVEL: It is an instrument which is used to establish horizontal line or surface.

- It consist of glass tube partially filled with liquid which should be stable, does not freeze at normal temp and must have low viscosity
ex: spirit, synthetic Alcohol, chloroforms.
- The space above liquid is occupied by vapour or air
- The longitudinal section of the tube is curved that is an arc of circle
- Graduations are marked over its upper surface which are equally spaced from each other (2mm)

Note: Since the curved surface is exactly circular equal distances on the tube subtends equal angle at the centre of the arc/curvature.

i.e $\theta = \frac{d}{R}$

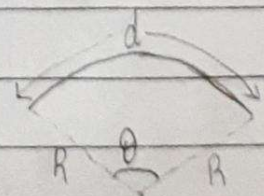


The longitudinal tangent to the curved surface at mid point represent the axes of tube

- when the bubble is centered the axes of tube is said to be horizontal axes and is perpendicular to horizontal axes

SENSITIVITY OF LEVEL TUBE:

It is defined as angle subtended at the centre of the arc of the level tube by one division of the tube in second.



$$\phi = \frac{d}{R} \text{ Radians}$$

$$\left\{ 1 \text{ Rad} = 206265 \text{ sec} \right\}$$

$$\phi = \frac{d}{R} \times \frac{180}{\pi} \times 60 \times 60$$

$$\phi = \frac{206265 d}{R} \text{ sec}$$

$$\text{if } d = 2 \text{ mm}$$

$$\phi = \frac{412.53}{R}$$

Note: The sensitivity is expressed as second / division

- This is not definite unless the length of division is also specified
- If not mentioned length of division may be taken as 2mm

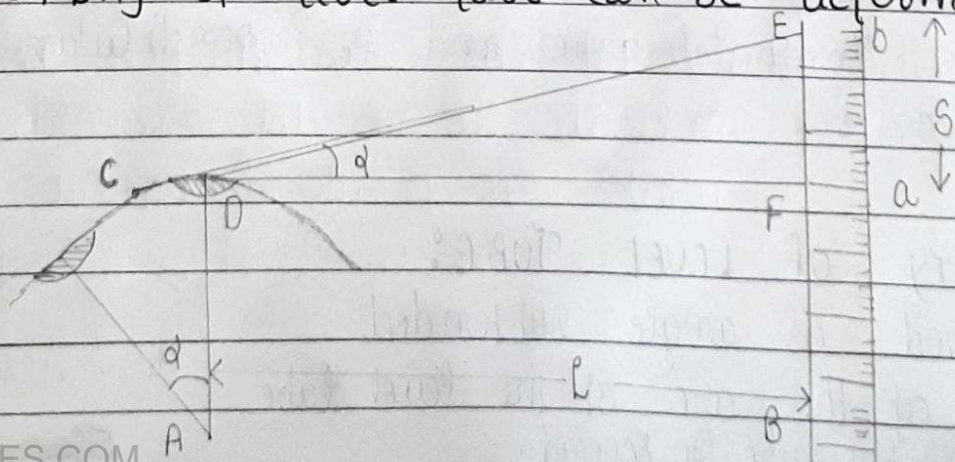
If the Bubble is at Centre by one division the tube axis will also make an angle of ϕ with the horizontal.

- sensitivity of level tube increases with

- (a) Increase in Radius (R)
- (b) decrease in viscosity
- (c) Increase in smoothness or decrease in Roughness of internal surface of tube
- (d) Increase in length of tube

Note: The sensitivity of tube must be compatible with the accuracy achieved with other components of tube.

Sensitivity of level tube can be determined as:



Difference b/w Readings on staff $S = b - a$
 Distance b/w Readings on level tube (CO) = md — (1)

$$\phi = \frac{d}{R}$$

In $\triangle ECF$

$$\tan d = \frac{S}{L} \quad \text{if } d \rightarrow \text{small}$$

$$d \rightarrow \tan d$$

$$d = \frac{S}{L} \quad \text{--- (2)}$$

$$\text{from level tube CO} = Rd \quad \text{--- (3)}$$

$$md = Rd$$

$$d = \frac{md}{R}$$

$$md = R \times \frac{S}{L} \Rightarrow \boxed{R = \frac{mdL}{S}}$$

$$\phi = \frac{d}{R} = \frac{d \cdot S}{mdL} = \frac{S}{Ln}$$

$$\boxed{\phi = \frac{S}{Ln}} \text{ Radian}$$

(Q) A Staff Reading taken on a staff held at a distance of 80m from the instrument with the Bubble Center was 1.455m. When the bubble was moved to 6 division out of the Center the staff Reading was observed to be 1.487m. If the length of 1 division is 2mm find the Radius of Curve and sensitivity of tube.

$$\text{Ans)} \quad L = 80m \quad S_1 = 1.455m \quad S_2 = 1.487m$$

$$R = \frac{mdL}{(S_2 - S_1)} = \frac{6 \times 2 \times 10^{-3} \times 80}{1.487 - 1.455} = 30m$$

$$\boxed{R = 30m}$$

$$\phi = \frac{S \text{ Rad.}}{Lm} = \frac{206265 \times (1.487 - 1.455)}{80 \times 6}$$

$$\phi = 13.75 \text{ s/division of 2mm.}$$

(Q) If the sensitivity of the bubble tube of a levelling instrument is 30" per 2mm division. Find the error in staff reading on a staff held at a distance of 10m, caused by a bubble being out of division from the center.

Ans)

$$\phi = 30'' \quad L = 10m$$

$$\phi = \frac{S}{Lm} \Rightarrow \phi S = 206265 \frac{S}{Lm}$$

$$S = \frac{30 \times 10 \times 2}{206265} = 0.0029m$$

Levelling is done with the help of unit termed as level which are generally of following types

(A) Dumpy level

(B) Tilting level

- In a dumpy level telescope and vertical spindle are rigidly casted as one piece.
- The levelling head consists of two plates where upper plate has telescope mounted over it and lower plate is attached to tripod.
- These plates are held apart with the help of 3-levelling screws.
- Before any reading is taken, the telescope has to be set horizontal in all directions.
- Due to the length of time required for levelling to other level it is not so popular.

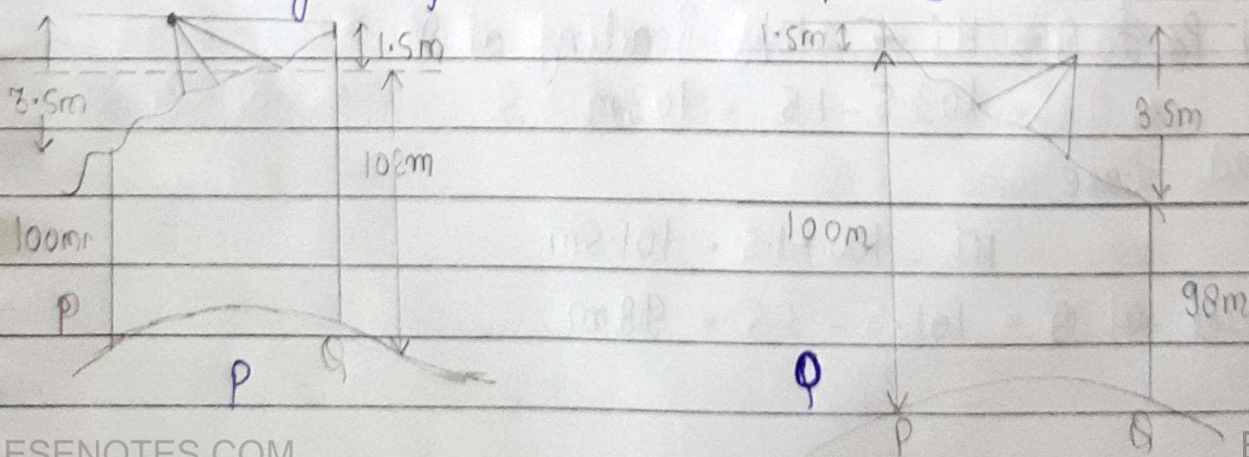
- In tilting level the telescope and vertical spindle are not locked rigidly with each other.
- In this case telescope can tilt slightly in vertical plane with the help of tilting screw.
- Here line of collimation is need not to be perpendicular to the vertical axis, as it was must in dumpy level.
- The line joining the point of intersection of cross hair and optical centre of the objective is called line of collimation.

Note: The levels are used to take the readings over the levelling staff which is a graduated rod of rectangular cross-section.

- These staffs are either 3m or 4m in length.
- Graduation on these staff gives the distance from the bottom.
- Each meter on the staff is divided into 200 divisions i.e. thickness of each division is 5mm, and these are painted white and black alternately.
- These staff are held vertical at the station under consideration which is checked with the help of bubble tube provided behind it.

PROCEDURE IN LEVELLING

The basic operation of levelling is to find difference in height of 2 concerned points.



So such that levelling staff of both stations are visible

For Example, 1st Case $\Delta h = 3.5 - 1.5 = 2\text{m}$

IInd Case $\Delta h = 1.5 - 3.5 = -2\text{m}$

- If the second reading is less than 1st one it indicates Rise
- If $\dots$ More $\dots$ fall
- If Actual level of any point is known, level of other points can also be computed.

- For Ex If level of Point P is 100m

1st Case, level of Point Q = level of Point P + Rise
 $= 100 + 2 = 102 \text{ m}$

$$= 100 + 2 = 102 \text{ m}$$

2nd Case, level of Point Q = level of Point P - Fall
 $= 100 - 2 = 98\text{m}$

$$z = 100 - 2 = 98 \text{ m}$$

These levels of points P and Q are termed as Reduced levels
As these are referred w.r.t to datum level.

As these are Referred w.r.t to datum level.

This method is termed as Rise and Fall method

- The same can be done by another method termed as height of collimation in which height of line of collimation is determined above the datum and is used further to find the level of other points

For Example :-

1st Case, ht of Collimation or Instrument Height (HI)

$$= \text{level of Point P} + \text{staff Reading at P}$$
$$HI = 100 + 3.5$$
$$= 103.5$$

Level of Point A = HI - staff Reading at A
 $= 103.5 - 1.5 = 102\text{m}$

$$= 103.5 - 1.5 = 102m$$

2nd Case

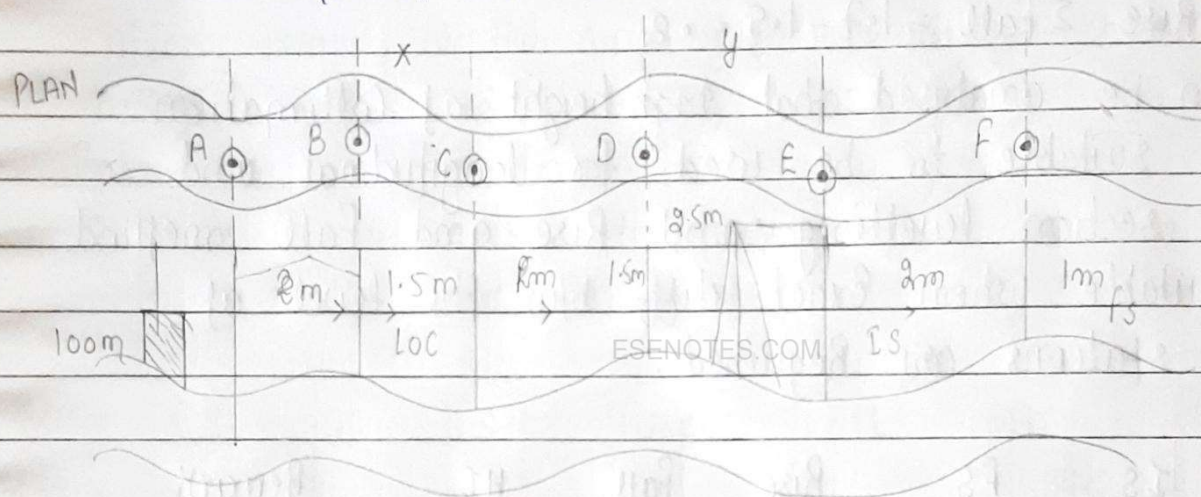
$$HI = 100 + 1.5 = 101.5m$$

level a) $Q = 101.5 - 3.5 = 98 \text{ m}$

The first Reading made with staff on a point whose RL is known is termed as BS

The last Reading made with staff on a point before changing the instrument station is termed as fore sight.

- All the Readings in between BS and FS made with staff is termed as Intermediate sight (IS)
- The station at which Position of level is changed is termed as change station.
- At this station Both BS and FS are observed.



BOOKING OF VALUES FOR LEVEL

While readings are booked in level Book which is specially designed for levelling Purpose

RL	BS	IS	FS	Rise	Fall	H.I	d	Remark
100	1m					101m	-	Bm
99m		2m			1m	101m	0	A
99.5m		1.5m		0.5m		101m	20	B
99m		2m			0.5m	101m	40	C
99.5m	2.5m		1.5m	0.5m		102m	60	D CP
100m		2m		0.5m		102m	80	E
100.2m			1.8	0.2		102m	100	F

once levelling is completed certain Arithmetic check are applied over the Reading

$$(1) \sum BS - \sum FS = \text{Last RL} - \text{First RL}$$

Note: The above check is not capable of identifying the error in intermediate side Hence for that Another check is applied

$$\sum \text{Rise} - \sum \text{Fall} = \text{Last RL} - \text{First RL}$$

$$\text{Ex: } \text{Last RL} - \text{First RL}$$

$$100.2 - 100 = 0.2$$

$$\sum B.S - \sum F.S = 3.5 - 3.3 = 0.2$$

$$\sum \text{Rise} - \sum \text{Fall} = 1.7 - 1.5 = 0.2$$

Note: It can be analysed that high height of collimation method is suitable to be used for longitudinal and cross-section levelling and Rise and Fall method is suitable when exact diff b/w the level of two stations are Required.

RL	BS	IS	FS	Rise	Fall	HI	Remark
560.500	0.865					561.365	A
559.260	1.025		2.105		1.240	560.285	B [change point]
558.705		1.580			0.555		C
558.420	2.230		1.865		0.285	560.650	D [change point]
557.815	2.255		2.835		0.605	560.170	E [change point]
558.410			1.760	0.595			F

$$\sum = 6.475$$

$$\sum = 8.515$$

$$\sum = 0.595$$

$$\sum = 2.685$$

Check:

$$\sum BS - \sum FS = \text{Last RL} - \text{First RL}$$

$$\sum \text{Rise} - \sum \text{Fall}$$

$$6.475 - 8.515 = 558.410 - 560.5$$

$$0.595 - 2.685$$

$$2.090 = 2.090$$

$$2.090$$

Note: Inverted staff: when the point whose Elevation is to be determine is above the Height of Collimation
 Ex. underside of beam arches, lintel

- In such cases staff is placed in Inverted Position with its zero end at Concern Point
- This Reading on Inverted staff is booked in appropriate Column of the level book with Negative sign.

Question): The following staff reading were observed successively with a level of instrument having been moved after third, sixth and eight readings: 2.228, 1.606, 0.988, 2.090, 2.864, 1.262, 0.602, 1.982, 1.044, 2.684 meters. Enter the above readings in a page of a level book and calculate RL of Points if the first reading was taken with the staff held on a bench mark of 432.384m

RL	BS	IS	FS	Rise	Fall	HI	Remark
432.384	2.228					434.612	Bm
433.006		1.606		.622			
433.624	2.090		.988	.618		435.714	CP1
432.85		2.864			.774		
434.452	.602		1.262	1.602		435.057	CP2
433.072	1.044		1.982		1.38	434.116	CP3
431.432			2.684		1.64		
$\Sigma = 5.964$			$\Sigma = 6.916$	$\Sigma = 2.842$	$\Sigma = 3.794$	Checked =	
			- 5.964		2.842		
			.952		.952		

Question) The field level book reading from a fly level are as follow:-

Find out the missing values (?) and also perform the arithmetic chain.

Date.....

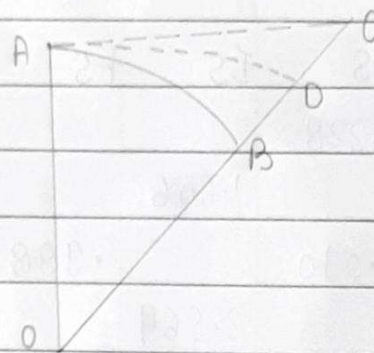
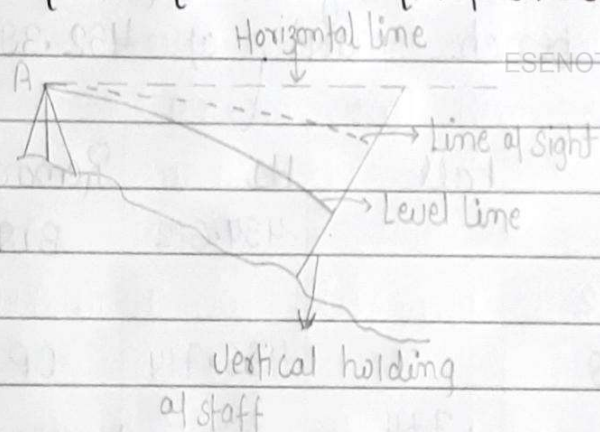
RL	BS	IS	FS	Rise	Fall	HI	Remark
100	3.635		-			103.635	Bm ₁
101.26	3.922		2.875	1.26		105.18	A (CP)
104.15	4.220		1.030	2.89		108.37	B (CP)
106.65	3.99		1.72	2.5		110.64	C (CP)
108.00			1.34	2.65			Bm ₂ (CP)
	$\Sigma = 15.765$		$\Sigma = 8.065$	$\Sigma = 8$	$\Sigma = 0$		

$$\Sigma BS - \Sigma FS = \text{First RL} - \text{Last RL} = \text{Rise} - \text{fall}$$

$$15.765 - 8.065 = 100 - 108 = 8 - 0$$

$$8 = 8 = 8$$

CURVATURE AND REFRACTION



$$OC^2 = OA^2 + AC^2, \quad \angle A = 90^\circ$$

Let $BC = C_c = \text{Correction for Curvature}$

$AB = d = \text{horizontal distance between A and B}$

$AO = R = \text{Radius of Earth [unit is same for all]}$

$$(R + C_c)^2 = R^2 + d^2$$

$$R^2 + C_c^2 + 2R \cdot C_c = R^2 + d^2$$

$$C_c (2R + C_c) = d^2$$

$$C_c = \frac{d^2}{2R + C_c} \approx \frac{d^2}{2R}$$

(Neglecting C_c in Comparison to $2R$)

$$C_c = \frac{d^2}{2R} = \frac{d \times 10^3}{2 \times 6370}$$

$$C_c = 0.078d^2 \text{ (m)}$$

Hence Correction due to Curvature

$$C_c = -0.0785d^2$$

The Effect of Curvature of Earth and Refraction Comes into Picture when sight distances are long.

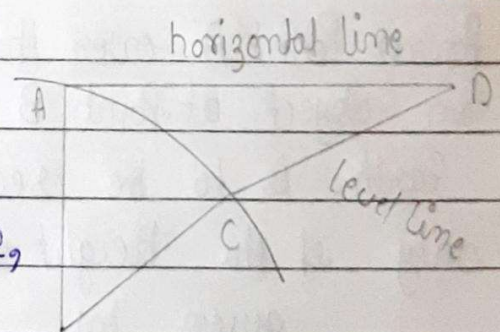
- This Effect occurs Because the difference between level line increase with distance.
- The horizontal line is tangential to the level surface at a point, whereas the level line is normal to the direction of Gravity at all the points.
- Hence while taking the observation from telescope the staff Reading x is obtained, However the level line has Reading y there by there is Positive Error $(C_c) - x - y$ Due to Curvature of Earth, Hence the Correction to be applied is negative which is Computed As above.

12) Effect due to REFRACTION

The density of air decreases ~~what~~ with increase in height above the surface of Earth.

- As the air is denser near the Earth surface, light from staff travel from

lighter medium to denser medium, Hence gets deflected ~~due~~ towards the normal. There by Reading obtained over the staff is comparatively smaller (C_d) than actual Hence Error is negative in this case and Correction is Positive.



- Correction due to Refraction is taken to be $1/7$ of the Correction due to Curvature of Earth

$$C_r = \frac{1}{7} C_c = \frac{0.0785 d^2}{7}$$

$$C_r = -0.0112 d^2 \text{ (Km)}$$

If Combined Correction due to both is Considered

$$C = C_c + C_r = -0.0785 d^2 + 0.0112 d^2$$

$$(m) \quad C = -0.06728 d^2 \text{ (Km)}$$

Note: In Case of longer sight Error due to Curvature of Earth, Refraction Error Can be Eliminated by Keeping the Instrument at Equal distance from station where BS and FS is taken

- As in this case nature and magnitude of Correction is same for both BS and FS
- If it is not-Possible to set up the Instrument in b/w the station in that Case Reciprocal of Levelling is being Carried out

DISTANCE OF VISIBLE HORIZON

from Point A over the surface of Earth if an object at Point B over the surface of Earth is to be seen it is Possible only if the height of the object is given by

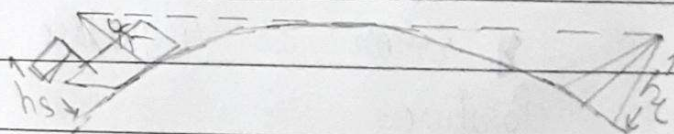
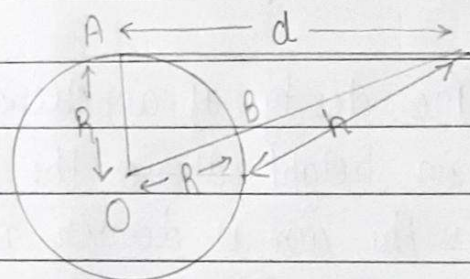
$$h = C_c = 0.0673 d^2$$

$$d = \frac{\sqrt{C_c/h}}{\sqrt{0.0673}} = 3.85 \sqrt{h} \text{ (m)}$$

Note: If both Point of observation are at Elevation in such case

$$D = d_1 + d_2$$

$$= 3.85 \sqrt{h_1} + 3.85 \sqrt{h_2}$$



$$D = 3.85 (\sqrt{h_s} + \sqrt{h_i}) \quad [D = \text{km}, h_s, h_i = \text{m}]$$

Note: In the previous analysis the height h of the objective under consideration was assumed to be much smaller than the Radius of the Earth But if it is not

$$R^2 + d^2 = (R + C_c)^2$$

$$R^2 + d^2 = R^2 + C_c^2 + 2R \cdot C_c$$

$$d = \sqrt{C_c^2 + 2RC_c}$$

$$\text{Correction due to Curvature} = -C_c$$

$$\text{Refractive} = -C_c/h$$

$$\text{Combined} = -6C_c/h$$

$$C_c' = \frac{-6C_c}{h}$$

Q $\Rightarrow$ A level was set up at Point C at a distance of 100m from A and 100m from B. The staff reading on the staff kept at A was 0.445m and that on staff B was 2.845m find the true difference in Elevation of A and B

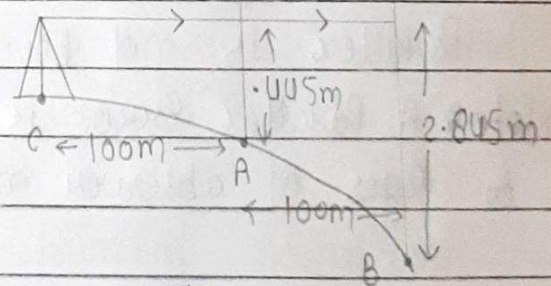
Sol^m

$$C_{CA} = -0.0673 d^2$$

$$= -0.0673 (1)^2$$

$$= -6.73 \times 10^{-4} \text{m}$$

$$\begin{aligned} \text{Correct Reading of staff at A} &= 0.445 - 6.73 \times 10^{-4} \\ &= 0.4443 \text{m} \end{aligned}$$



$$C_{CB} = 0.0673 (d^2) = -0.0673 (1)^2 = -0.0673 \text{m}$$

$$\text{Corrected Reading of staff at B} = 2.845 - 0.0673 = 2.777$$

Corrected diff of Elevation B/w A and B

$$= 2.777 - 0.443$$

$$= 2.33 \text{m}$$

Date.....

Q) An observer standing on the deck of ship just sees the top of a light which is 40m above the MSL. If the height of the observer eye is 8m above the MSL determine the distance ^{b/w} the ship and the light house.

Soln)

$$D = d_1 + d_2$$

$$D = 3.85 (\sqrt{40} + \sqrt{8})$$

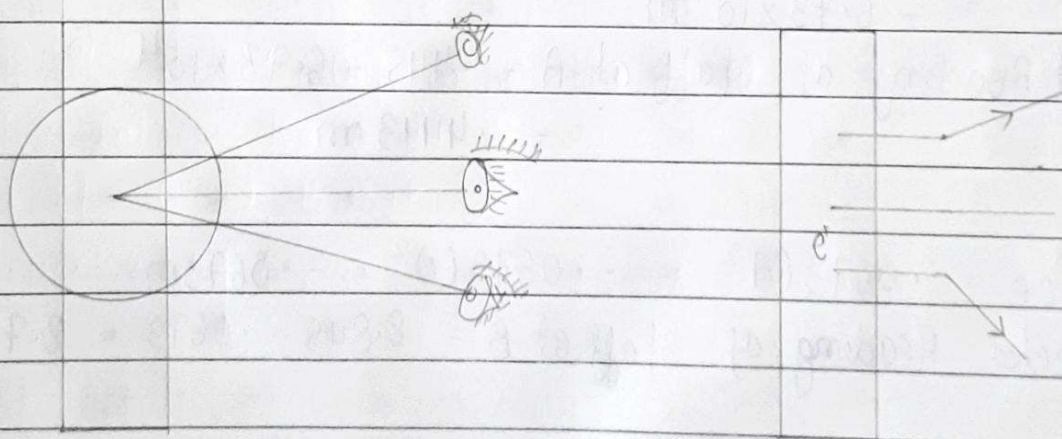
$$D = 35.24 \text{ Km}$$

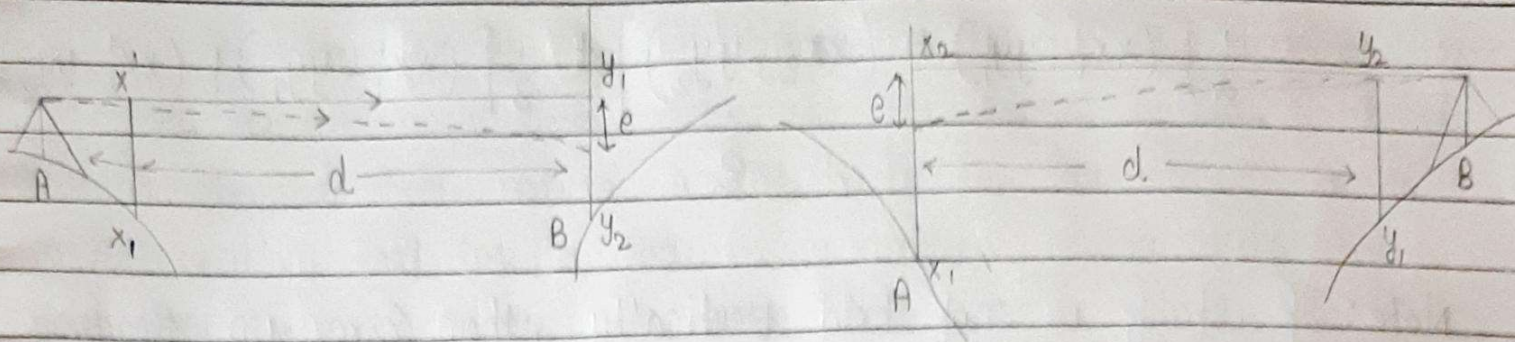
RECIPROCAL LEVELLING

Reciprocal levelling is used to determine the correct difference in the elevation of two points which are at large distance from ~~two points~~ each other and it is not possible to set up the instrument at mid way of both these points station.

For Ex - To find difference in elevation of two points on opp. side of river or valley. It eliminates the effect of curvature of earth, refraction and collimation but ~~these~~ effect due to Parallax is not being eliminated.

Note: A Parallax Error is An Error in Reading ~~in~~ of instrument due to eyes of observer not in plane perpendicular to the plane of scale.





Level at A

apparent diff b/w Elevation of A & B
 $= x_1 - y_1$

True diff = $x_1 - (y_1 - e)$

True diff = A and B = app diff
 b/w Elevation A & B + e (1)

Level at B

app diff b/w A & B = $x_2 - y_2$

True diff = $(x_2 - e) - y_2$

True diff b/w Elevation of A & B

= app diff b/w
 A & B - e (2)

from (1) and (2)

True diff b/w Elevation A and B = $\frac{1}{2}$ [sum of app diff]

$e = \frac{1}{2}$ [change of app diff]

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$e = e_{\text{error}} + e_c + e_{\text{coll}}$

$e = e_c + e_d$

$e_d = e - e_c$

$e_d = e - 0.0673d^2$

e = Total Error

e_r = Error due to refraction

e_c = Error due to Curvature

e_{coll} = Error due to Collimation

e_{cc} = Combined Error due to Curvature
 and Refraction

Note: In the above Analysis Two similar Instrument adjusted to same extent are being used at both the station. This method not only corrects error due to Curvature and Refraction, but also due to adjustment of Collimation axis, but in this case if temperature changes error due to refraction also changes.

If two Instrument are used but are not adjusted to same extent in such case, they should be interchanged after taking first reading. In such case true diff b/w Elevation of A & B

$$= \frac{1}{2} [(xx_1 - yy_1) + (xx_2 - yy_2)] + \frac{1}{2} [(xx'_1 - yy'_1) + (xx'_2 - yy'_2)]$$

Note: If staff is not held vertically, then Error in Reading is given by

$$\text{Error}(e) = AB' - AB$$

$$\cos \theta = \frac{AB}{AB'}$$

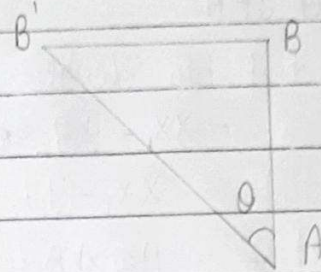
$$AB' = AB \sec \theta$$

$$e = AB (\sec \theta - 1)$$

Note: Permissible error in ordinary levelling

$$e = \pm 24\sqrt{K}$$

K = Permissible Error of traverse



Ques) The following observation were taken in Reciprocal levelling

Instrument at	Staff Reading	A	B
A		1.625	2.545
B		.75	1.405

Ans) True difference in Elevation = $\frac{1}{2} [(1.625 - 2.545) + (.75 - 1.405)]$
 of A and B
 $= -.8$

$$R_L \text{ of } B = R_L \text{ of } A - .8 = 101 - .8 = 100.2 \text{ m}$$

Remaining Part of Question: Determine the R_L of B if that of A is 101m

- Also Calculate the angular Error in Collimation if distances b/w A and B is 1km

$$(a) \text{ Error} = \frac{1}{2} [(.725 - 1.405) - (1.625 - 2.545)]$$

$$= .12$$

$$e_d = e - 0.0673d^2 = .12 - 0.0673(1)^2 = .0527$$

$$\tan d = \frac{e_d}{d} \Rightarrow d = \tan^{-1}\left(\frac{e_d}{d}\right) = \tan^{-1} \frac{.0527 \times 60 \times 60}{1 \times 10^3}$$

$$d = 10.87''$$

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Question) If the sensitivity of the bubble tube of a level is 30 sec. of Arc per division of 2mm, determine the distance of a point at which Combined Curvature and Refraction Correction becomes numerically equal to the error induced by dislevelment of one division of level in level tube.

Ans) $\theta = 30 \text{ sec}$ $d = 2 \text{ mm}$

$$C_c = C_R$$

$$\theta = \frac{206265}{L} \text{ S}$$

$$C_c = -0.673d^2$$

$$L \text{ m}$$

$$\frac{206265}{L} = 0.673d^2$$

$$206265$$

$$\frac{206265}{206265} = \frac{0.673 \times 10^6}{206265}$$

$$L = \frac{1 \times 0.673 \times 10^6}{206265}$$

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$$\frac{206265}{206265} = \frac{0.673 \times 10^6}{206265}$$

$$206265 \times 0.673 \times 10^6$$

$$\frac{30 \times 10^3}{206265} = 0.673L^2$$

$$206265$$

$$L = \frac{30 \times 10^3}{0.673 \times 206265}$$

$$0.673 \times 206265$$

$$L = 2161 \times 10^3 \text{ km}$$

$$L = 2161 \text{ m}$$

TACHEOMETRY / TACHYMETRY

- It is defined as optical distance measurement method
- In Comparison to chaining or taping on flat ground its Accuracy is ~~more~~^{less}, and on steep surface, its accuracy is more
- It is done with the help of a device known as tachemetry
- Tachemeter is a transit theodolite fitted with stadia diaphragm.
- Stadia diaphragm consist of two stadia hairs at equal distance one above and one below the horizontal hair of cross hair

Note: Tachemeter Posses special characteristic

- It has multiplication Constant $K=100$
- Additive Constant = 0
- Telescope of Tachemeter has analytic lens fitted on it.
- Magnification Power of Eyepiece is kept high

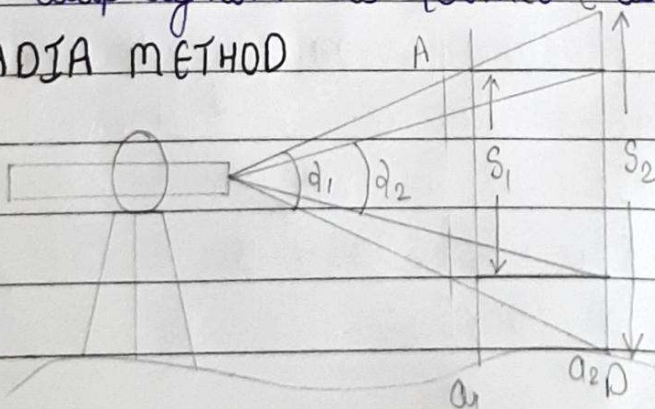
Note: Up to the distance of 100m, normal levelling staff can be used in tachemetry, but beyond that distance marking on the staff becomes indistinct hence in such cases stadia Rod is used.

METHODS OF TACHEOMETRY

Tachemetry Can be done by any of the following method.

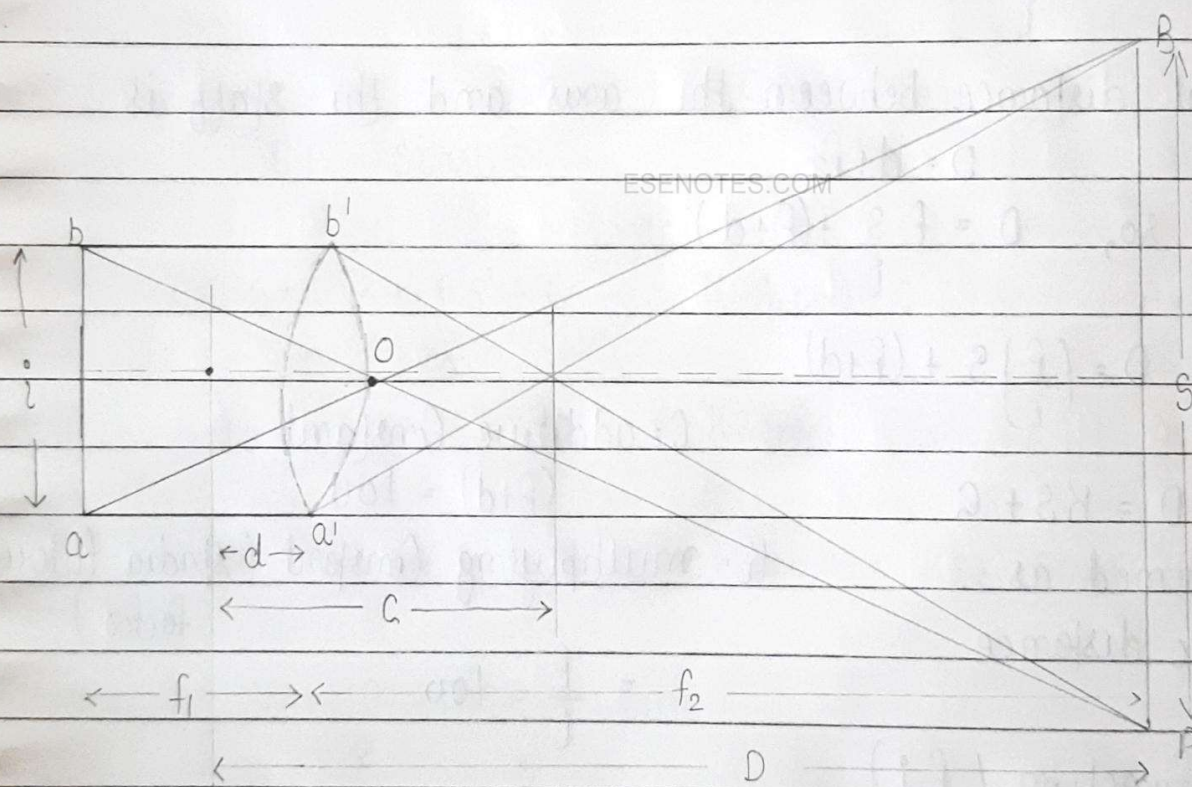
- Additional hair being provided other than cross hair in diaphragm is termed as stadia hair.

(I) STADIA METHOD



- Vertical distance between the two stadia hairs is termed as stadia Interval and angle made by them is termed as stadia/tacheometer/Parallactic Angle.
- when the stadia Angle (d) and staff Intercept is varied from (S_1 to S_2) this type of stadia Intercept is termed as fixed hair mro.
- However if the stadia Angle is varied (from d_1 to d_2) the staff Intercept is kept Constant (S_1), this type of stadia method is termed as movable Hair/substance method
[use to determine short distance upto 200m] [use when surface is flat]

PRINCIPAL OF STADIA METHOD



$ab = i$ = Interval between the stadia hairs (stadia Interval)

$AB = S$ = staff Intercept

f = focal length of objective

f_1 = Horizontal distance of the cross wires from O

f_2 = Horizontal distance of the staff from the optical of the object.

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D = horizontal distance of the staff from the vertical axis of instrument

Since $\triangle aob \cong \triangle AOB$

$$\frac{f_1}{f_2} = \frac{i}{S} \quad \text{--- (1)}$$

from lens maker formula

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Multiply both side by $f f_2$

$$f_2 = f \frac{f_2}{f_1} + f \quad \text{--- (2)}$$

$$f_2 = f \frac{S}{i} + f \quad \text{Put value of (1) in (2)}$$

horizontal distance between the axis and the staff is

$$D = d + f_2$$

$$\text{so, } D = f \cdot \frac{S}{i} + (f + d)$$

$$D = \left(\frac{f}{i} \right) S + (f + d)$$

$$D = KS + C$$

C = additive Constant

$$(f + d) = 100$$

K = multiplying Constant (stadia Interval factor)

$$= \frac{f}{i} = 100$$

This Eqⁿ is termed as
tachometric distance
Equation

K = Function (f, i)

For ease of Calculation of D , stadia Interval i is adjusted
that $K = f/i$ become equal to 100.

Since the value of almost Remain Constant for
tachometer " C " is also Constant

In General $C = 0.25 - 0.35$

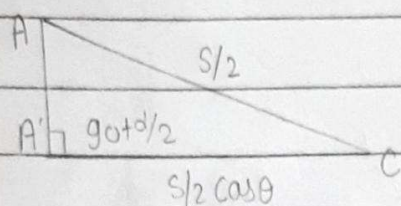
If analytic lens is used $C = 0$

$$\boxed{D = KS}$$

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(i) FIXED HAIR METHOD (Angle of Elevation)

(1) Staff Vertical



$$\angle COA = \theta \quad \angle A'OB' = d$$

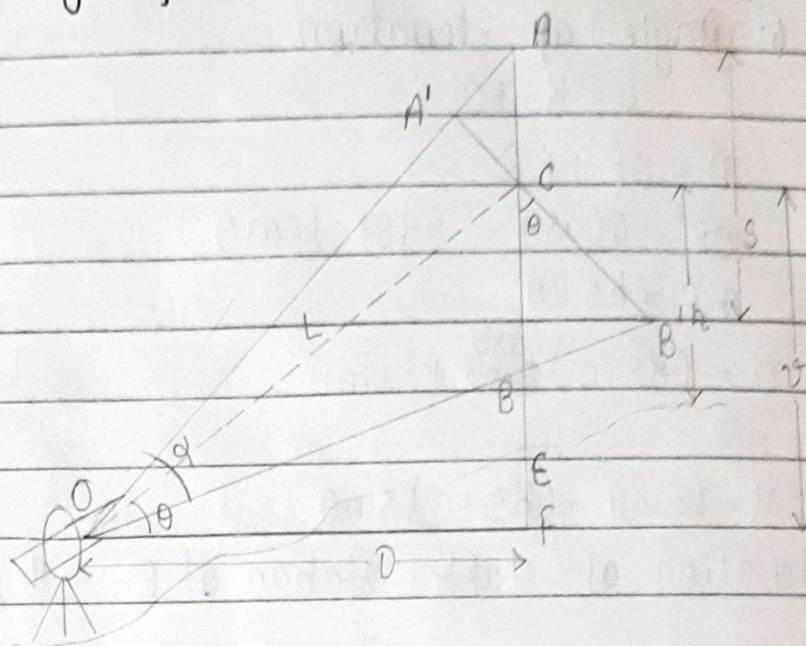
$$\angle OCA = 90 - \theta \quad \angle A'OC = d/2$$

$$\angle OCB_1 = 90 \quad \angle OCA' = 90^\circ$$

$$\angle BCB_1 = \theta \quad \angle OA'C = 90 - d/2$$

$$\angle AA'C = 180 - (90 - d/2)$$

$$\angle AA'C = 90 - d/2$$



$$A'B' = A'C + B'C = S/2 \cos \theta + S/2 \cos \theta$$

$$A'B' = S \cos \theta$$

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$$L = Ks + C = K A'B' + C = K \cdot S \cos \theta + C$$

$$D = L \cos \theta$$

$$D = (K \cdot S \cos \theta + C) \cos \theta = K S \cos^2 \theta + C \cos \theta$$

 θ = Vertical angle d = Stadia angle

OC = line of sight

$$V = FC = L \sin \theta = (K S \cos \theta + C) \sin \theta = K S \sin \theta \cos \theta + C \sin \theta$$

$$V = \frac{KS}{2} \sin 2\theta + C \sin \theta$$

$$\text{Elevation of staff station } E = \text{Height of Instrument} + V - h$$

(ii) Staff Vertical angle of depression

$$\text{Elevation of staff station} = \text{Height of Instrument} - V - h$$

(b) Staff Normal

(b) Angle of Elevation

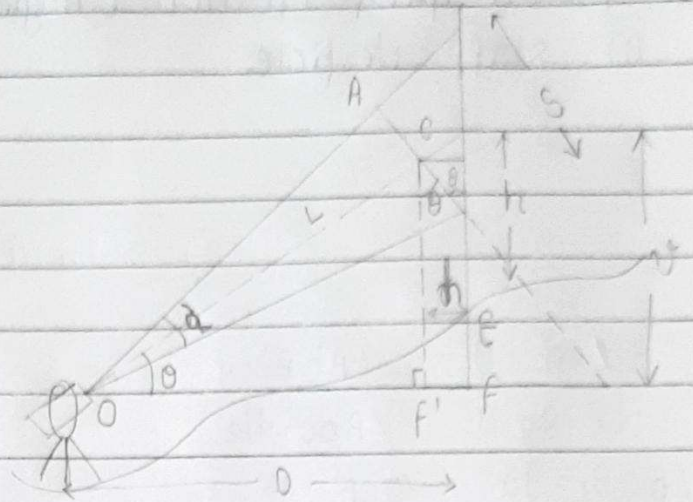
$$L = KS + C$$

$$0 = 0f' + f'f$$

$$OF' = L \cos \theta = (Ks + c) \cos \theta$$

$$f'f = h \sin \theta$$

$$0 = (Ks + c) \cancel{\sin \theta} + h \sin \theta$$



$$V = L \sin \theta = (kg + c) \sin \theta$$

Elevation of staff station at E = Height of Instrument + V - h_{case}

(ii) Angle of depression

$$D = L \cos \theta - h \sin \theta$$

$$V = (Ks + c) \sin \theta$$

$$D = (Ks + c) \cos \theta - h \sin \theta$$

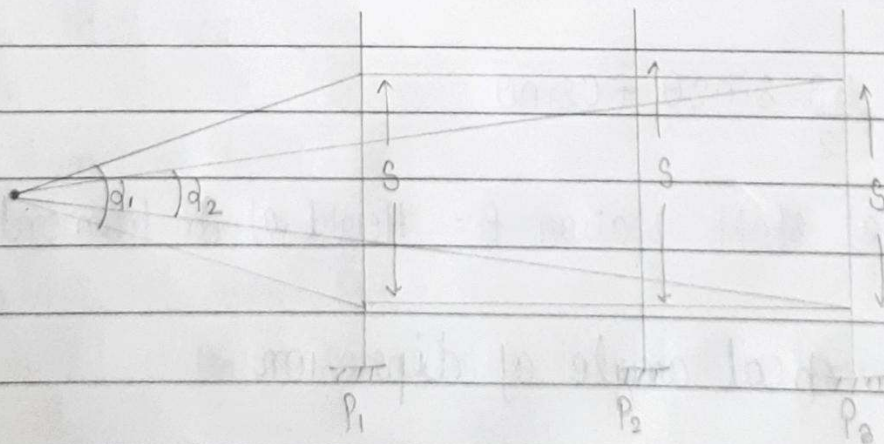
Elevation of staff station

$$P = \text{Height of Instrument} - (v + h_{\text{case}})$$

$$= HI - v - h_{\text{case}}$$

$$= H \sin \theta - v - h \cos \theta$$

(2) MOVABLE HAIR METHOD : In this case staff intercept is kept constant 's' and stadia angle (α) changes with staff position (P)



- The diaphragm has an arrangement for the measurement of stadia interval which in this case is variable.
- Each hair of stadia diaphragm can be moved independently by a separate sliding bar or frame.

- sliding frame consist of a micrometer screw which rotates over large horizontal graduated scale or Head

$$D = \frac{f}{i} S + (f+d)$$

- The stadia Interval i is measured with the help of micrometer screw for the staff intercept S having pitch P
- let m be the total number of the revolution of the micrometer screw for the staff intercept S . Then

$$i = mp$$

$$D = \frac{f}{mp} S + (f+d) \quad \text{or} \quad D = \frac{KS}{m} + C$$

where $K = \frac{f}{P}$ Constant for an instrument

C = additive Constant

Case I: If staff is held vertical and line of sight is inclined

$$D = \frac{KS \cos^2 \theta}{m} + C \cos \theta$$

$$V = \frac{1}{2} \frac{KS \sin 2\theta}{m} + C \sin \theta$$

Case II: If staff is held normal and line of sight is inclined

$$D = \left(\frac{KS}{m} + C \right) \cos \theta + h \sin \theta$$

$$V = \left[\frac{KS}{m} + C \right] \sin \theta$$

Note: - If there is index error " e " in the micrometer screw then

$$D = \frac{KS}{m-e} + C$$

Refraction error is least

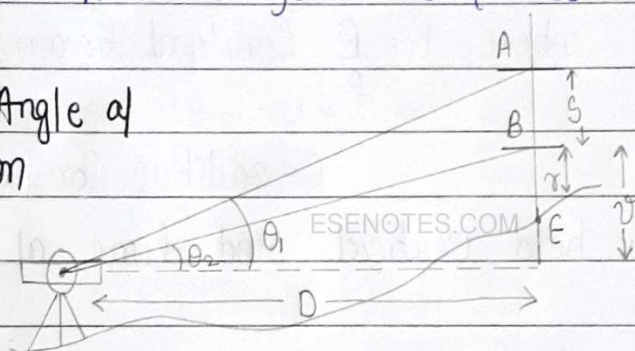
- This method gives more accurate Result for long distance measurement, but it is very slow, and noting of stadia Reading is difficult in this case and Computation are complicated

#(3) TANGENTIAL METHOD of TACHEOMETRY

This method is used when diaphragm is not provided with stadia hair or when distance b/w staff and instrument is more which in this method, levelling staff is fitted with two big target (Vanes) at a fixed distance of 2m or 3m

- Vertical Angle of these vanes are observed through the theodolite to find horizontal distance as follow.

Case 1) Both are Angle of Elevation



$$\tan \theta_1 = \frac{V+s}{D} \quad \text{--- (1)} \quad , \quad \tan \theta_2 = \frac{V}{D} \quad \text{--- (2)}$$

from (1) and (2)

$$\tan \theta_1 - \tan \theta_2 = \frac{s}{D}$$

$$D = \frac{s}{\tan \theta_1 - \tan \theta_2}$$

RL of staff station E = Height of station + V - v

Case (2) Both are Angle of depression.

$$\tan \theta_1 = \frac{V-s}{D} \quad \text{--- (1)} \quad \tan \theta_2 = \frac{V}{D} \quad \text{--- (2)}$$

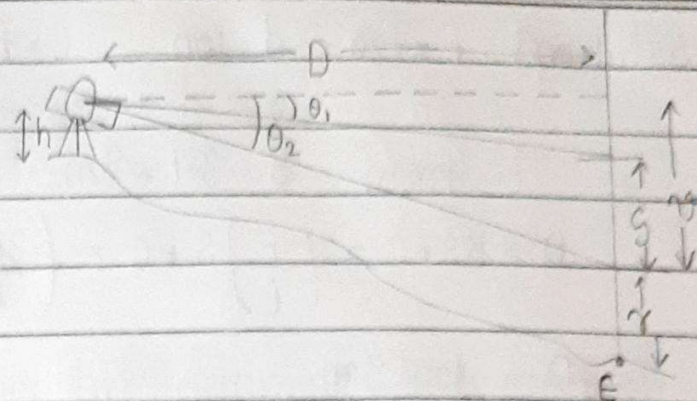
from (1) and (2)

$$\tan \theta_2 - \tan \theta_1 = \frac{S}{D}$$

$$D = \frac{S}{\tan \theta_2 - \tan \theta_1}, [V = D \tan \theta_2]$$

RL of Instrument station

$$E = \text{Height of Instrument} - [V + R]$$

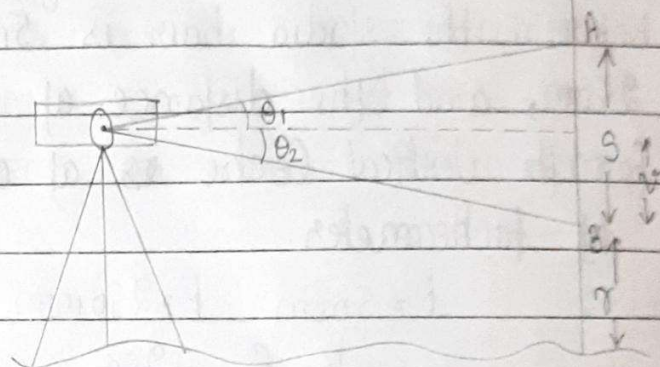


Case 3: one of the Angle is Elevation and other is depression.

$$\tan \theta_1 = \frac{S-V}{D} \quad (1) \quad \tan \theta_2 = \frac{V}{D} \quad (2)$$

from (1) + from (2)

$$\tan \theta_1 + \tan \theta_2 = \frac{S}{D}$$



$$D = \frac{S}{\tan \theta_1 + \tan \theta_2} [V = D \tan \theta_2]$$

This method is inferior to stadia as

- (1) Time Required is more due to two angles being noted
- Error may ensue if the instrument is disturbed in between two readings
- while taking the reading the atmosphere condition may get change inducing error due to Refraction.

Question) The focal length of a object glass of a tachometer is 200mm. The distance between the Vertical Axes of tachometer and the optical Center of the object glass is 100mm and the spacing b/w the upper and lower stadia is 4mm. while the line of collimation perfectly horizontal the staff readings are 1m top, 2m middle and 3m bottom. The horizontal in (m) between the staff and the instrument station is in meter.

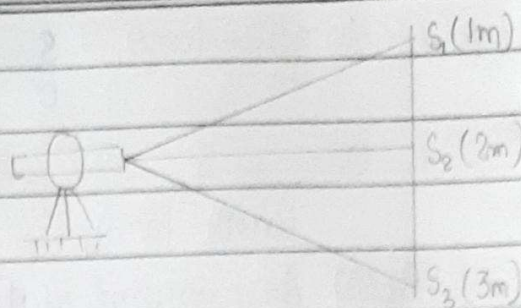
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Solution} $F=200$ $d=100$ $C=F+d=200+100$
 $=300\text{mm}$

$i=4\text{mm}$ $S=3-1=2\text{m}$

$D = Ks + C = \left(\frac{F}{i}\right)s + C = \left(\frac{200}{4}\right) \times 2 + 3$

$D = 100.9\text{m}$



Question) In an External focusing telescope tachometer, the fixed interval between the stadia hairs is 5mm the focal length of the objective is 250mm, and the distance of the vertical axes of the instrument from the vertical centre of objective is 150mm. Find the Constant of tachometer.

Ans): $i=5\text{mm}$ $F=250\text{mm}$ $d=150\text{mm}$

$C = F + d = 250 + 150$
 $C = 400\text{mm}$

$K = \frac{F}{i} = \frac{250}{5} = 50\text{mm}$

$K=50\text{mm}$, $C=400\text{mm}$

Question) The following observation were taken with a tachometer at a station P to a staff at Q held normal to the line of sight
 Staff Reading = 1.45, 1.915, 2.380m

Angle of depression = $15^\circ 30'$

RL of P = 201.45

Height of instrument axis above the Peg at P = 1.35

Determine Horizontal distance b/w P and Q and RL of Q.

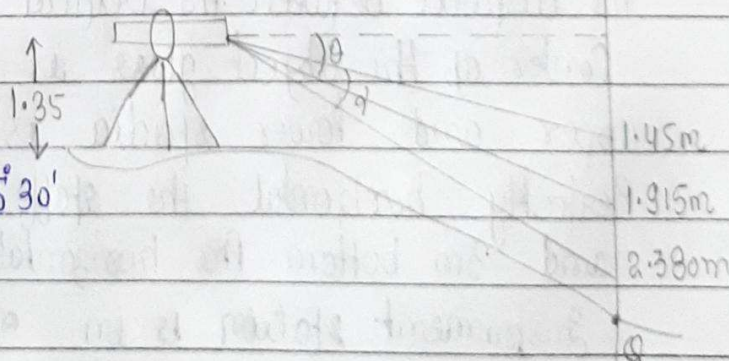
assume $K=100$, $C=0$.

Solution) $D = L \cos \theta - h \sin \theta$

$D = (Ks + C) \cos \theta - h \sin \theta$

$D = (100 \times 0.93) \cos 15^\circ 30' - 1.915 \sin 15^\circ 30'$

$D = 89.105\text{m}$



$$\begin{aligned}
 RL \text{ of } Q &= HI - [V + h \cos \theta] \\
 &= RL \text{ of } P + \text{Turnion axes} - [L \sin \theta + h \cos \theta] \\
 &= 201.45 + 1.315 - [1.93 \sin 15^\circ 30' + 1.915 \cos 15^\circ 30'] \\
 &= 176.066 \text{ m}
 \end{aligned}$$

Question) A substance theodolite was used to determine the horizontal distance of a point from the instrument station. The micrometer readings of the drum & diaphragm were respectively 3.425 and 3.93 when the staff intercept was 3m. The micrometer screw has 100 thread to 1cm. The focal length of the object glass was 225mm. The distance of the instrument axes from the object glass was 200mm.

~~Ans~~ solution) $f = 225 \text{ mm}$ $d = 200 \text{ mm}$

$$\text{Pitch } P = 1/100$$

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$$K = \frac{f}{P} = \frac{22.5}{1/100}$$

$$K = 2250$$

$$C = f + d = 225 + 200 = 425 \text{ mm}$$

$$C = 0.425 \text{ m}$$

$$S = 3 \text{ m}$$

$$m = 3.425 + 3.93$$

$$m = 7.355$$

$$D = \frac{KS}{m} + C$$

$$D = \frac{2250 \times 3}{7.355} + 0.425 = 918.16 \text{ m}$$

Question) Staff Reading observed with a Percentage Theodolite corresponding to Angle of Elevation of 4% and 5% are 1.525 and 2.925 respectively. If the Vertical Angle on sighting the staff Reading equal to the height of turnion axes above ground was 4.5%. Calculate the horizontal distance b/w instrument and staff.

(ii) The Elevation of staff station if that of the instrument station was 500mm.

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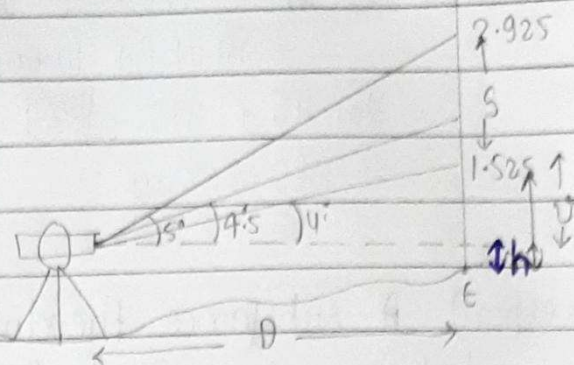
solution)

$$\tan 5^\circ = \frac{V+S}{D}$$

$$\tan 4^\circ = \frac{V}{D}$$

$$\tan 5^\circ - \tan 4^\circ = \frac{S}{D} \quad [5.84 \text{ or } 1\%]$$

$$D = \frac{1.4}{\tan 5^\circ - \tan 4^\circ} = \frac{140 \text{ mm}}{0.05 - 0.04}$$



$$S = 2.925 - 1.525$$

$$S = 1.4$$

RL of staff instrument

$$= \text{Height of station} + V - h$$

$$= 500 + 5.57 - 2.225$$

$$\Rightarrow 500 + 2.225 + 5.6 - 1.525 = 506.3 \text{ m}$$

$$\text{So, } h = v + v' = 1.525 + 0.7 = 2.225$$

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$$\tan 4^\circ = \frac{V}{D}$$

$$V = 5.57 \text{ m}$$

$$D = \frac{S'}{\tan \theta_3 - \tan \theta_2}$$

$$S' = 140 [\tan 5^\circ - \tan 4^\circ]$$

$$S' = 0.7$$

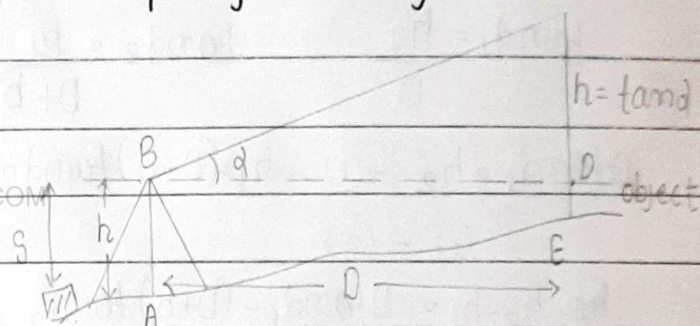
$$S = 0.7$$

TRIGONOMETRIC LEVELLING

- It is Indirect method of levelling in which the Relative Elevation of various points are determined from vertical angle and horizontal distance (D) measured with a theodolite and tape/chain / Tacheometer Respectively.
- It is not as accurate as direct levelling, but can be used for topographic work or where direct levelling is not possible
- It can be done as follow

Case-I: Determination of level at the top of an object when its base is accessible.

$$\begin{aligned} \text{RL of Point C at top of object} \\ &= \text{RL of line of Collimation} + h \\ &= \text{RL of BM} + S + h \end{aligned}$$



Case II: Determination of level at top of an object when its base are accessible

In ΔCGD

$$\tan \alpha_1 = \frac{h}{D}$$

$$h = D \tan \alpha_1 \quad \text{--- (1)}$$

In ΔCFD

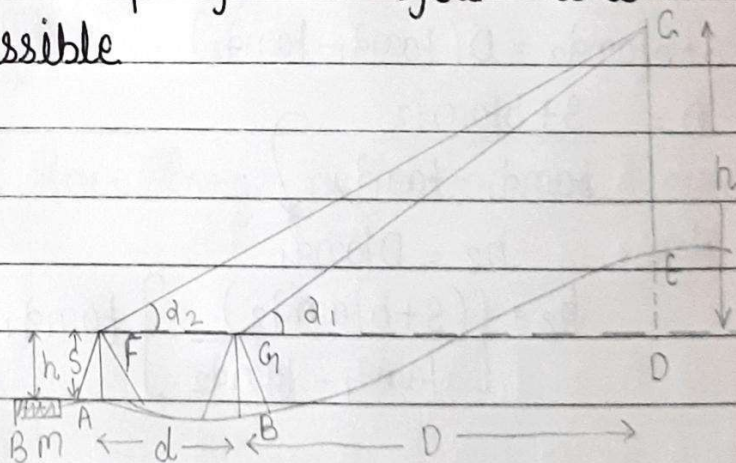
$$\tan \alpha_2 = \frac{h}{D+d}$$

$$h = \tan \alpha_2 (D+d)$$

$$D (\tan \alpha_1 - \tan \alpha_2) = d \tan \alpha_2$$

$$D = \frac{d \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}$$

$$\tan \alpha_1 - \tan \alpha_2$$



$$D = \frac{d \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}$$

$$h = D \tan \alpha_1$$

$$h = \frac{d \tan \alpha_2 \cdot \tan \alpha_1}{\tan \alpha_1 - \tan \alpha_2}$$

RL of top point of object C

$$= \text{RL of I.C.} + h$$

$$= \text{RL of BM} + s + h$$

$$= \text{RL of BM} + s + \frac{d \tan \alpha_2 \tan \alpha_1}{\tan \alpha_1 - \tan \alpha_2}$$

Case b: Instrument at station A is higher than at B

In $\Delta G O D_1$

$$\tan \alpha_1 = \frac{h_2}{D}$$

In $\Delta F O D_2$

$$\tan \alpha_2 = \frac{h_1}{D+b}$$

$$D \tan \alpha_1 = h_2 \quad (1) \quad h_1 = (D+b) \tan \alpha_2 \quad (2)$$

$$(1) - (2)$$

$$h_2 - h_1 = D \tan \alpha_1 - (D+b) \tan \alpha_2$$

$$S = D \tan \alpha_1 - b \tan \alpha_2 - D \tan \alpha_2$$

$$S = D (\tan \alpha_1 - \tan \alpha_2) - b \tan \alpha_2$$

$$S + b \tan \alpha_2 = D (\tan \alpha_1 - \tan \alpha_2)$$

$$D = \frac{S + b \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}$$

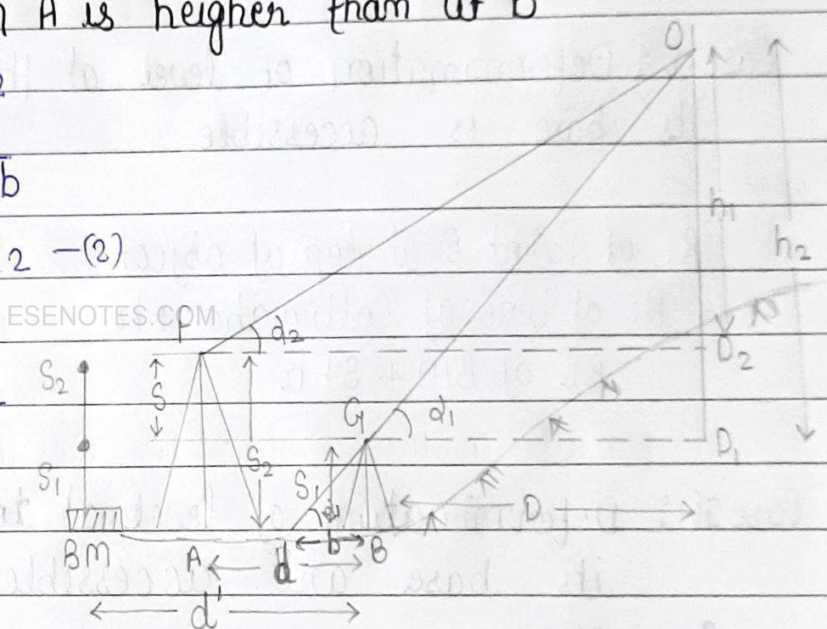
Now, $h_2 = D \tan \alpha_1$

$$h_2 = \left[\frac{(S + b \tan \alpha_2)}{\tan \alpha_1 - \tan \alpha_2} \right] \tan \alpha_1 = \left[\frac{(S \cot \alpha_2 + b) \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \right] \tan \alpha_1$$

$$= \frac{(S \cot \alpha_2 + b) \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \quad \text{or} \quad \frac{(b + S \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)}$$

RL of top point (C) object = RL of line of collimation at station B or A + h_1 or h_2

$$\text{RL of Top Point C} = \text{R.L. of BM} + s_1 \text{ or } s_2 + h_1 \text{ or } h_2$$



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In $\Delta G_1 L H$ ~~Case 1: $S =$~~

$$\tan \alpha_1 = \frac{S}{b} \Rightarrow b = S \cot \alpha_1$$

In $\Delta O G_1 D_1$ In $\Delta O M O_2$

$$\tan \alpha_1 = \frac{H_1}{D} \quad (1)$$

$$\tan \alpha_2 = \frac{H_2}{D+d} \quad (2)$$

$$(2) - (1) \Rightarrow$$

~~Case 2:~~

$$H_2 - H_1 = \tan \alpha_2 (D+d) - \tan \alpha_1 D$$

$$S = D \tan \alpha_2 + d \tan \alpha_2 - D \tan \alpha_1$$

$$S = D (\tan \alpha_2 - \tan \alpha_1) + d \tan \alpha_2$$

$$\frac{S - d \tan \alpha_2}{(\tan \alpha_2 - \tan \alpha_1)} = D$$

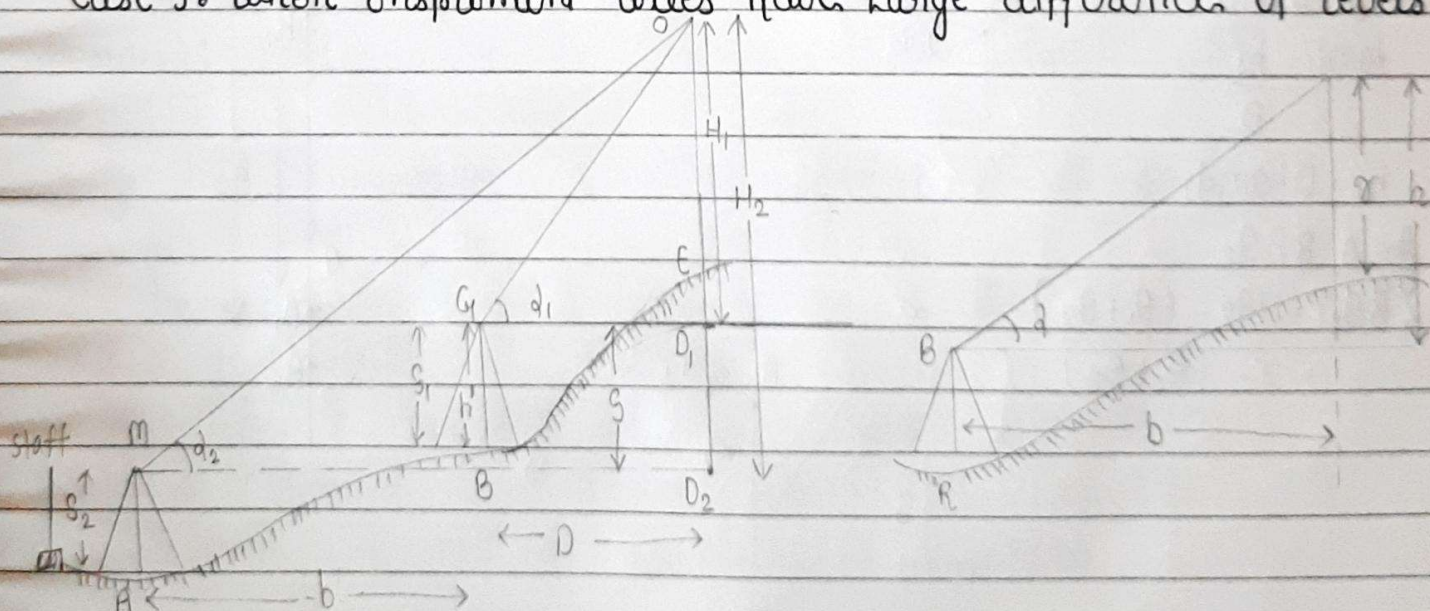
$$D = \frac{(S \cot \alpha_2 - d) \tan \alpha_2}{\tan \alpha_2 - \tan \alpha_1}$$

$$\text{Now, } H_1 = D \tan \alpha_1$$

$$H_1 = \left[\frac{(S \cot \alpha_2 - d) \tan \alpha_2}{\tan \alpha_2 - \tan \alpha_1} \right] \tan \alpha_1$$

$$H_1 = \frac{(S \cot \alpha_2 - d) \tan \alpha_1 \tan \alpha_2}{\tan \alpha_2 - \tan \alpha_1}$$

Case 3: When Instrument axes Have Large difference of levels



$$h_1 = D \tan \alpha_1 \quad (1) \quad h_2 = (D+d) \tan \alpha_2 \quad (2)$$

$$(2) - (1)$$

$$h_2 - h_1 = (D+d) \tan \alpha_2 - D \tan \alpha_1$$

$$D = \frac{d \tan \alpha_2 - S}{\tan \alpha_1 - \tan \alpha_2}$$

$$D = \frac{(d - S \cot \alpha_2) \tan \alpha_1}{\tan \alpha_1 - \tan \alpha_2}$$

$$h_1 = \frac{(d - S \cot \alpha_2) \tan \alpha_2 \tan \alpha_1}{(\tan \alpha_1 - \tan \alpha_2)}$$

Ht of station B above line of Collimation of Instrument at station A = $h - i = b \tan \alpha - i$

Height of axis at B above the axis at A = $S = b \tan \alpha - i + h'$

$h' =$ ht of Instrument at station B

RL of top Point of object at $P_1^0 =$ RL of B + h_1

$$= \text{RL of A} + S + h_1$$

$$= \text{RL of A} + S + h_1 \quad \text{where } S = b \tan \alpha - i + h'$$

Case 4: Determination of Height of object when the two Instrument are not in same Vertical plane

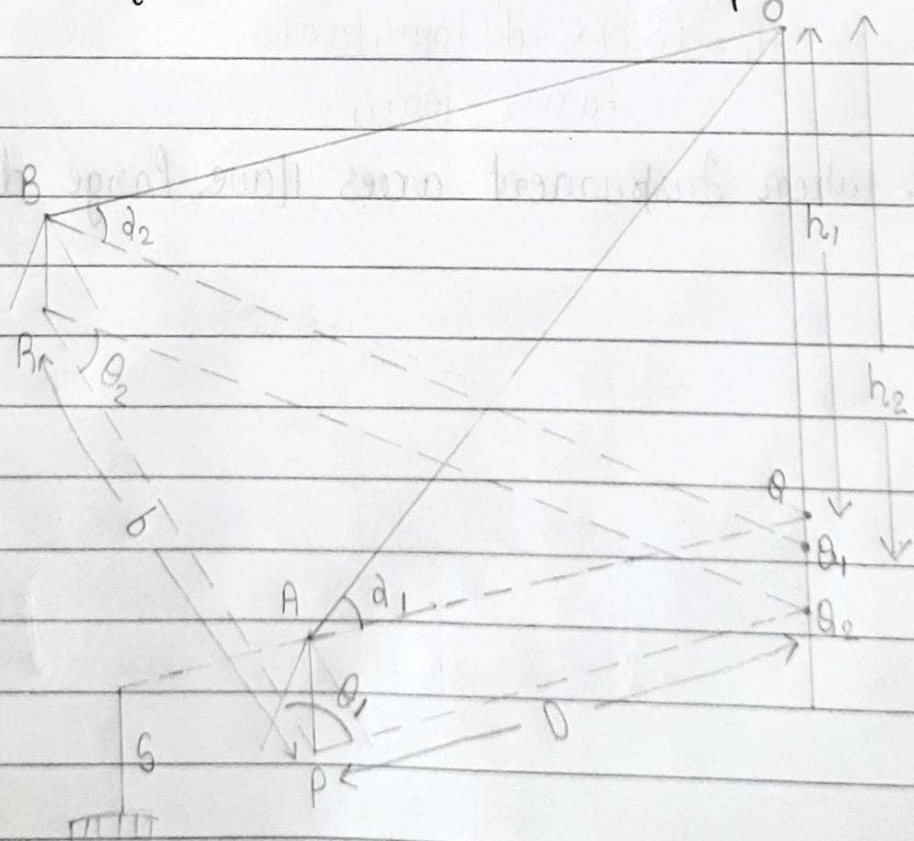
In $\triangle OAB$
 $\tan \alpha_1 = \frac{h_1}{D}$

$$h_1 = D \tan \alpha_1$$

In $\triangle RPQ_2$

$$\angle RPQ_2 = 180 - (\theta_1 + \theta_2)$$

$$= \pi - (\theta_1 + \theta_2)$$



By sine rule

$$\frac{PO_2}{\sin \theta_2} = \frac{RO_2}{\sin \theta_1} = \frac{RP}{\sin [\pi - (\theta_1 + \theta_2)]}$$

$$\frac{PO_2}{\sin \theta_2} = \frac{RO_2}{\sin \theta_1} = \frac{b}{\sin (\theta_1 + \theta_2)}$$

$$PO_2 = D = \frac{b \sin \theta_2}{\sin (\theta_1 + \theta_2)} \quad \text{and} \quad RO_2 = \frac{b \sin \theta_1}{\sin (\theta_1 + \theta_2)}$$

$$\text{Now, } h_1 = D \tan \theta_1$$

$$h_1 = \frac{b \sin \theta_2 \tan \theta_1}{\sin (\theta_1 + \theta_2)}$$

$$R.L \text{ of } O = R.L \text{ of } BM + S + h_1$$

$$\text{also } h_2 = \frac{b \sin \theta_1 \tan \theta_2}{\sin (\theta_1 + \theta_2)}$$

similarly

Question) Find the R.L of a church spire C from the following observation taken from a station A and B 50 m apart

Angle BAC = 60° , Angle ABC = 50° , Angle of Elevation from A to the top of spire = 30° and Angle of

ee : ee B ee ee ee ee = 29°

staff Reading from A on BM (20) = 2.5

ee ee B ee ee (20) = 0.5 m

Answer): $b = D = 50$, $\theta_1 = 60^\circ$, $\theta_2 = 50^\circ$, $\alpha_1 = 30^\circ$, $\alpha_2 = 29^\circ$, $S_1 = 2.5 \text{ m}$, $S_2 = 0.5 \text{ m}$

$$h_1 = \frac{b \sin \theta_2 \tan \alpha_1}{\sin (\theta_1 + \theta_2)} = \frac{50 \sin 50^\circ \tan 30^\circ}{\sin (60^\circ + 50^\circ)} = 23.53 \text{ m}$$

$$h_2 = \frac{b \sin \theta_1 \tan \alpha_2}{\sin (\theta_1 + \theta_2)} = \frac{50 \sin 60^\circ \tan 29^\circ}{\sin (60^\circ + 50^\circ)} = 25.54$$

$$R.L \text{ of } C \text{ from } A = R.L \text{ of } BM + S_1 + h_1 = 20 + 2.5 + 23.53 = 46.03$$

$$R.L \text{ of } C \text{ from } B = R.L \text{ of } BM + S_2 + h_2 = 20 + 0.5 + 25.54 = 46.042$$

$$R.L \text{ of } C = (46.03 + 46.042) = 46.036 \text{ m}$$

One of the objective of many ~~surveyor~~ ^{survey} series is to measure Area and Volume

The method adopted for measurement of Area and Volume depends on

(A) Accuracy required

(B) Shape or Geometry of triangle

- Measurement can be done by either

(A) Field measurement

(B) Plan measurement

COMPUTATION OF AREA OF GEOMETRICAL FIGURE

1) Rectangle = $L \times B$

2) Square = B^2

(3) Circle = $\frac{\pi D^2}{4}$

(4) Triangle = $\frac{1}{2} \times B \times H$

(5) Parallelogram = $B \times H$

(6) Trapezium = $\frac{1}{2} [B_1 + B_2] \times H$

(7) Regular Polygon = $\frac{1}{2} \times \text{perimeter} \times R$, [R = Perpendicular distance of any side from the

n = no of sides

L = length of one side

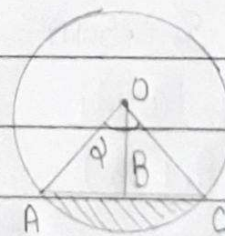
Center or $\frac{nL^2 \cot(\frac{180}{n})}{4}$]

(8) oblique triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
 $s = \frac{a+b+c}{2}$

(9) Area of segment

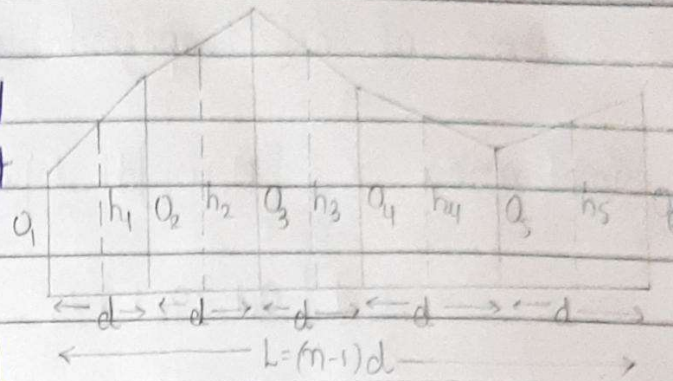
$\Delta ABC = \frac{\pi D^2 \alpha}{4 \times 360} - \frac{D^2 \sin \alpha}{8}$

$\Delta AC = \frac{1}{2} A \times h = \frac{1}{2} \left[2 \cdot \frac{D \sin \frac{\alpha}{2}}{2} \cdot \frac{D \cos \frac{\alpha}{2}}{2} \right]$
 $= \frac{D^2 \sin \alpha}{8}$



#(A) MID-ORDINATE RULE

Let $h_1, h_2, \dots, h_{(n-1)}$ be the mid ordinate of diff segment



$$A = A_1 + A_2 + A_3 + A_4 + \dots + A_{(n-1)}$$

$$A = h_1 d + h_2 d + h_3 d + h_4 d + \dots + h_{(n-1)} d$$

$$A = [h_1 + h_2 + h_3 + \dots + h_{(n-1)}] d$$

$$A = \frac{[h_1 + h_2 + h_3 + \dots + h_{(n-1)}] d (n-1)}{(n-1)}$$

$A = \text{average of all mid ordinate of all segment} \times \text{Length of base}$

#(B) Average ORDINATE METHOD

$A = \text{Average of all the ordinate} \times \text{Length of base}$

$$A = \frac{(O_1 + O_2 + \dots + O_n)(n-1)d}{n}$$

or

$$\frac{(n-1)d [\sum O_i]}{n}$$

or

#(C) TRAPEZOIDAL METHOD.

The Accuracy of this method is more than mid ordinate and avg. ordinate method. This method is also known as Parabolic Rule or End rule.

- The End ordinate are considered in this case though they can equal to zero

Here area is given by

$$\text{Area of 1st trapezoidal} = \frac{1}{2} [O_1 + O_2] d$$

$$\text{or } \text{Area of 2nd trapezoidal} = \frac{1}{2} [O_2 + O_3] d$$

$$\text{Area of last trapezoidal} = \frac{1}{2} [O_{n-1} + O_n] d$$

$$\text{Total area} = \frac{1}{2} [O_1 + O_2] d + \frac{1}{2} [O_2 + O_3] d + \dots + \frac{1}{2} [O_{n-1} + O_n] d$$

$$= \frac{1}{2} d [O_1 + 2O_2 + 2O_3 + \dots + 2O_{n-1} + O_n]$$

$$A = d \left[\left(\frac{O_1 + O_n}{2} \right) + O_2 + O_3 + O_4 + \dots + O_{n-1} \right]$$

(D) SIMSON'S ONE THIRD RULE / PRISMATOIDAL RULE

If the boundaries of irregular tract are curved than this method is preferred over trapezoidal method due to its Higher Accuracy.

- In this case the curved boundary is considered to be a parabolic arch.

$$A = \text{Area of } ABHG_1F$$

$$= \text{Area of } ABHG_1 + \text{Area of } BDH_1EB$$

$$\text{Area of } ABHG_1 = \frac{1}{2} [O_1 + O_3] 2d$$

$$\text{Area of } BDH_1EB = \frac{2}{3} \text{Area of } \text{triangle } BCDH$$

$$= \frac{2}{3} \times DE \times 2d$$

$$= \frac{2}{3} \left[O_2 - \left[\frac{O_1 + O_3}{2} \right] \right] \times 2d$$

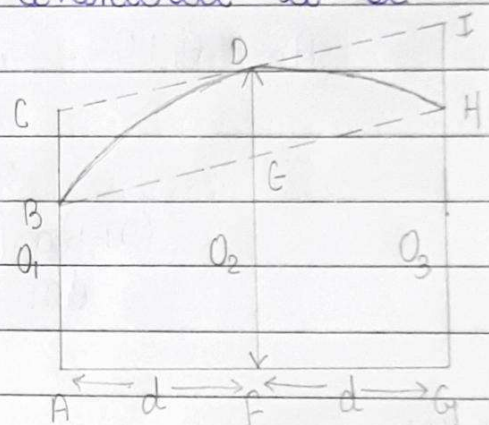
$$DF = O_2$$

$$DE = DF - EF$$

$$EF = \frac{O_1 + O_3}{2}$$

$$A = \frac{2d}{3} \left[2O_2 - (O_1 + O_3) \right]$$

$$\text{Total area} = \frac{1}{2} [O_1 + O_3] 2d + \frac{2d}{3} [2O_2 - (O_1 + O_3)]$$



$$= \frac{[3(O_1 + O_3)2d + 4d[2O_2 - O_1 + O_3]]}{6}$$

$$\Rightarrow \frac{d}{6} [6O_1 + 6O_3 + 8O_2 - 4(O_1 + O_3)]$$

$$A = \frac{d}{6} \left[\frac{3(O_1 + O_3)2}{2} + 4[2O_2 - (O_1 + O_3)] \right]$$

$$A_1 = \frac{d}{3} [O_1 + 4O_2 + O_3]$$

Area of next two segment

$$A_2 = \frac{d}{3} [O_3 + 4O_4 + O_5]$$

Area of last two segment

$$A_n = \frac{d}{3} [O_{n-1} + 4(O_{n-1} + O_n)]$$

Area of Trapezium $A = A_1 + A_2 + \dots + A_n$

$$h = \frac{d}{3} [O_1 + 4O_2 + O_3] + \frac{d}{3} [O_3 + 4O_4 + O_5] + \frac{d}{3} [O_{n-2} + 4O_{n-1} + O_n]$$

$$A = \frac{d}{3} [(O_1 + O_n) + 4(O_2 + O_4 + O_6 + \dots + O_{n-1}) + 2(O_3 + O_5 + O_7 + \dots + O_{n-2})]$$

$$\text{Area} = \frac{d}{3} [\text{Sum of 1st and last ordinate} + 4 \times [\text{Even}] + 2 \times [\text{odd ordinate}]]$$

Note: This Rule is applicable when there is odd no of offsets or even no of segment.

- If there are even No. of offsets or odd no of segment, then also this Rule can be applied but into two ~~parts~~ Parts

(A) Area upto 2nd last segment is computed using Simpson's rule

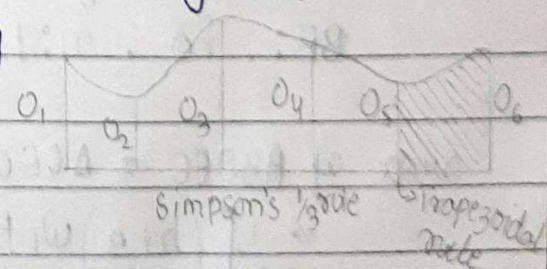
- If in this case problem demands computation

of area using Simpson's Rule only. But

It has even no of offsets and odd

no of segments in such case take

the avg of any two adjacent offset hence no of offset will now be odd.



MEASUREMENT OF VOLUME

The Computation of the volume of different quantities is Required for planning and designing of various Engg. works.

- Embankment of highway, Railway, Retaining wall etc.
- Reservoirs Capacity
- Concreting works
- Storage requirement
- Silos, bunkers.

The direct volume measurement is Practically not possible in these Engg work [because of their size] Hence it is Computed as follow.

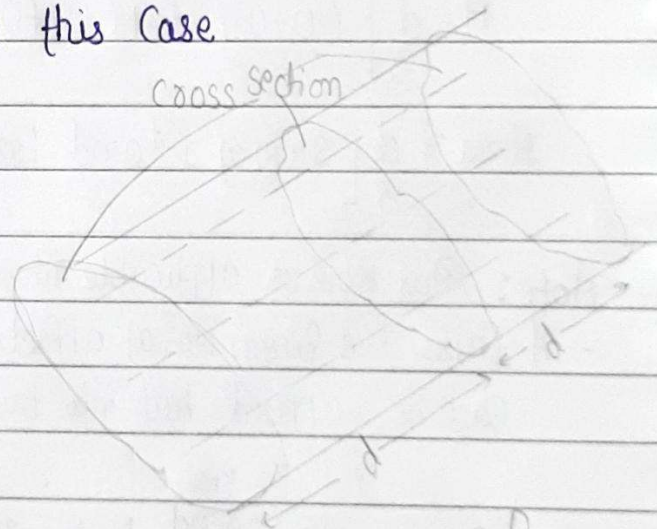
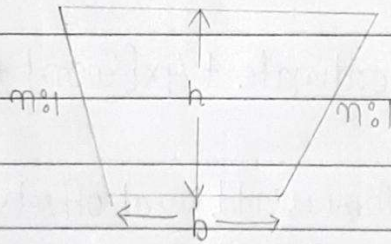
(A) MEASUREMENT OF VOLUME USING AREA OF CROSS-SECTION METHOD

It is generally used to find the volume of Earthwork in which cross-section are taken at right Angle to the fixed center line.

- spacing between cross-section depends upon on Accuracy required, ~~and~~ availability of cost, type of terrain
- Following types of c/s are taken in this case

$$A = \frac{1}{2} [b + b + 2mh] h$$

$$= [b + mh] h$$



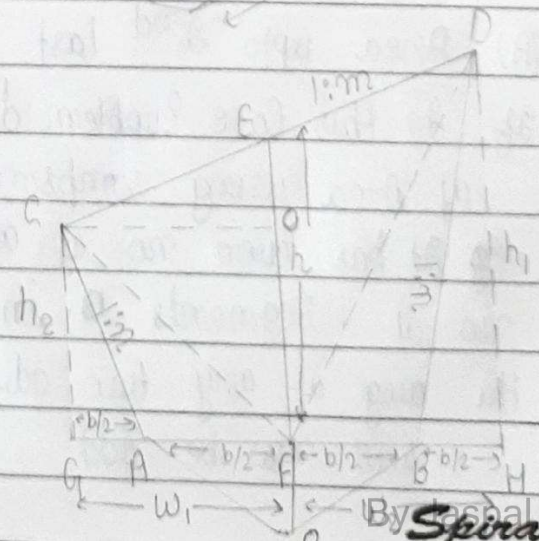
(B) Two level Section

$$BF :: FO :: n:1$$

$$FO = \frac{b}{2n}$$

$$\text{area of ABDEC} = \Delta CGO + \Delta EOD - \Delta ABO$$

$$= \frac{1}{2} \left[\left(\frac{h+b}{2n} \right) W_1 + \left(\frac{h+b}{2n} \right) W_2 - \frac{b^2}{2n} \right]$$



$$= \frac{1}{2} \left[(w_1 + w_2) \left(\frac{h+b}{2n} \right) - \frac{b^2}{2n} \right] \quad - (a)$$

$$\begin{aligned} \text{Area of } \Delta ABDEC &= \Delta CAF + \Delta DFB + CFE + \Delta DEF \\ &= \frac{1}{2} \left[\frac{b \cdot h_2}{2} + \frac{b \cdot h_1}{2} + hw_1 + hw_2 \right] \quad - (b) \end{aligned}$$

Now,

$$BH = n \cdot h_1$$

$$BH = FH - FB = w_2 - b/2$$

$$\therefore nh_1 = w_2 - b/2 \quad - (i)$$

$$\text{also, } w_2 = (h_1 - h)m \quad - (ii)$$

Put (ii) in (i)

$$nh_1 = (h_1 - h)m - b/2$$

$$h_1(n-m) = -mh - b/2$$

$$h_1(m-n) = mh + b/2$$

$$h_1 = \frac{mh + b/2}{(m-n)}$$

$$h_1 = \frac{m}{(m-n)} \left[\frac{h+b}{2m} \right] \quad - (iii)$$

Put h_1 in (i)

$$w_2 = \frac{b}{2} + \frac{mm}{(m-n)} \left[\frac{h+b}{2m} \right]$$

Similarly

$$h_2 = \frac{m}{(m+n)} \left[\frac{h-b}{2m} \right] \quad \text{and} \quad w_1 = \frac{b}{2} + \frac{mm}{m+n} \left[\frac{h-b}{2m} \right]$$

Put value in Eqn (a)

$$\text{Area} = \frac{m^2n}{m^2-n^2} \left[\frac{h+b}{2m} \right]^2 - \frac{b}{4m}$$

Put all values in Eqn (b)

$$\text{Area} = \frac{n \left(\frac{b}{2} \right)^2 + m^2 (bh + nh^2)}{(m^2 - n^2)}$$

(C) THREE LEVEL SECTION.

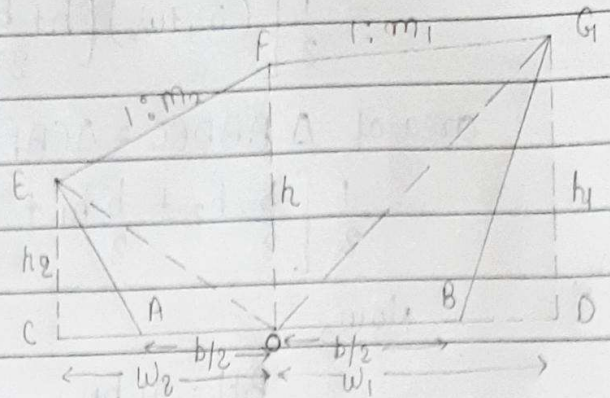
Similar way

$$w_1 = \frac{m_1 n}{m_1 - n} \left[\frac{h+b}{2n} \right]$$

$$w_2 = \frac{m_2 n}{m_2 - n} \left[\frac{h+b}{2n} \right]$$

$$h_1 = \left[\frac{h+w_1}{\frac{m_1}{m_1-n}} \right] = \frac{m_1}{m_1-n} \left[\frac{h+b}{2n} \right]$$

$$h_2 = \left[\frac{h-w_2}{\frac{m_2}{m_2+n}} \right] = \frac{m_2}{m_2+n} \left[\frac{h+b}{2n} \right]$$



area of ABGFE = ΔGAO + ΔGBO + ΔEFO + GFO

$$= \frac{1}{2} \left[\left(h_2 \times \frac{b}{2} \right) + \left(h_1 \times \frac{b}{2} \right) + h w_2 + h w_1 \right]$$

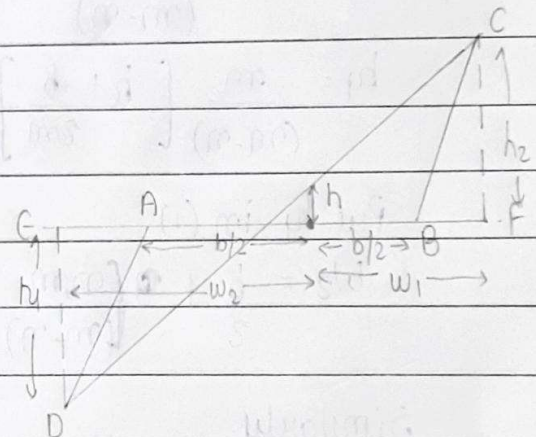
$$= \frac{1}{2} \left[\frac{b}{2} (h_1 + h_2) + \frac{h}{2} (h_1 + h_2) \right]$$

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(D) SIDE HILL TWO LEVEL SECTION

$$A_1 = \frac{1}{2} [b h_1] = \frac{1}{2} \left[\frac{b+m h}{2} \right] \left\{ \frac{m}{m-n} \left[\frac{h+b}{2n} \right] \right\}$$

$$A_2 = \frac{1}{2} [b h_2] = \frac{1}{2} \left[\frac{b-m h}{2} \right] \left\{ \frac{m}{m-n} \left[\frac{b-h}{2n} \right] \right\}$$



Volume of Earthwork Can be now Computed as follow

(A) TRAPEZOIDAL / END AREA METHOD

$$V = \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \dots + A_{n-1} \right] d$$

d = distance b/w consecutive Area of Cross-section

(2) PRISMATOIDAL / SIMPSON'S $\frac{1}{3}$ rd RULE

$$V = \frac{d}{3} \left[(A_1 + A_n) + 4(A_2 + A_4 + \dots + A_{n-1}) + 2(A_3 + A_5 + \dots + A_{n-2}) \right]$$

- It is necessary to have odd no of cross-section to apply this
- If even no of cross-section are present then volume is computed using Simpson rule upto odd cross-section and for the last segment volume is computed using end area method.
- In case if only Prismatoidal method is to be used the area halfway b/w the section is interpolated by averaging the dimension of any two consecutive section and not by averaging the Area section Method.

Note: For Estimation of volume of water in Reservoir the Contour map is used in this case

- volume of Earthwork, As in case of Foundation etc. "Spot level" of ground are being used it is also termed as unit area method.

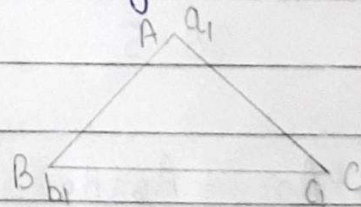
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In this method the entire area is divided into no of regular geometric figures and the level of the corners of this figure is noted before and after construction

$$V = \left[\frac{a_1 + b_1 + c_1}{3} \right] \times A$$

A = Area of fig.

A_1, b_1, c_1 are depth of excavation at corner of the fig.



Question: Excavation is to be made for Reservoir measuring 20m long, 12m wide at the bottom and 2m deep. The sides slopes are to be 1:1.2 And the top of the be flushed with the ground which is level in the vicinity. As per Prismatoidal Formula the volume of Excavation would be

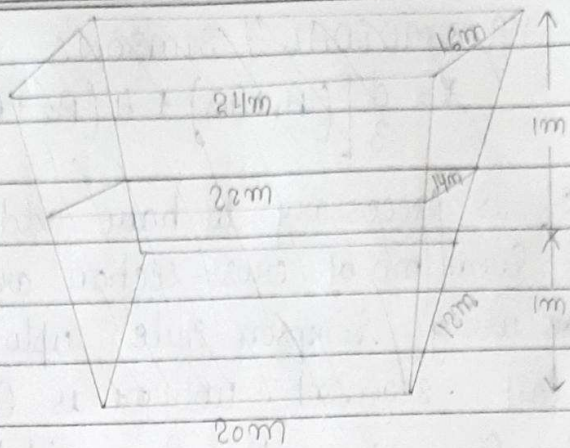
Date.....

Soln) $A_1 = 20 \times 12 = 240 \text{ m}^2$
 $A_2 = 22 \times 14 = 308 \text{ m}^2$
 $A_3 = 24 \times 16 = 384 \text{ m}^2$ $d = 1 \text{ m}$

$$V = \frac{d}{3} [A_1 + 4A_2 + 2A_3]$$

$$V = \frac{1}{3} [240 + 4 \times 308 + 2 \times 384]$$

$$V = 746.66 \text{ m}^3$$

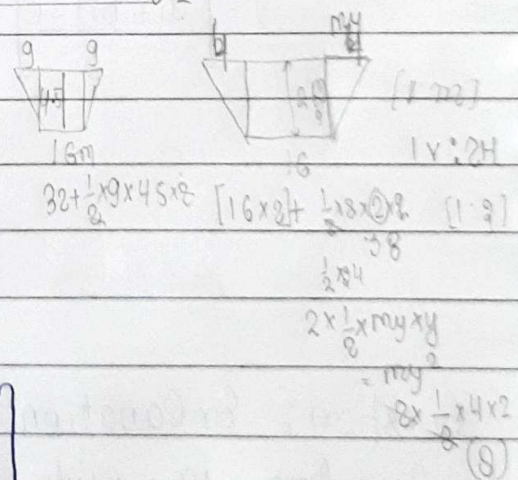
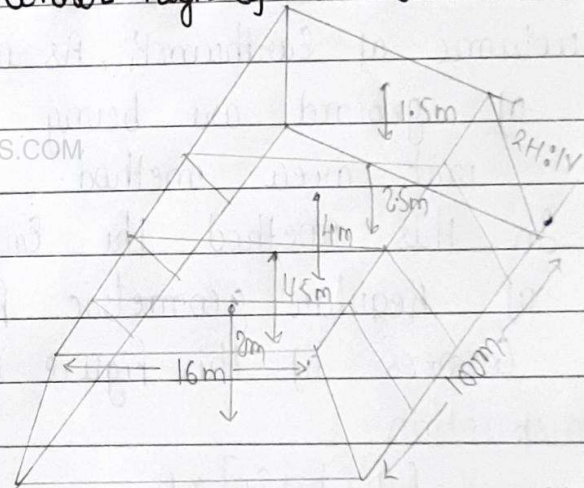


Question} A Railway Embankment is 16m wide with side slope 2H:1V. Assume the ground to be level in the direction traverse to the Centre line. Calculate the volume of Earthwork Contained in a length of ~~100m~~ ^{100m} using trapezoidal and Simsons method the Center High of 20m Interval 100m is 2, 4.5, 4, 3.5, 2.5, 1.5 Respectively.

Soln

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Point(x)	H	Area = $(bht + mh^2) \text{ (m}^2\text{)}$
0	2	$16 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 40$
20	4.5	$16 \times 2 + \frac{1}{2} \times 9 \times 4.5 \times 2 = 112.5$
40	4	96
60	3.5	80.5
80	2.5	52.5
100	1.5	28.5



$$V = d \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \dots + A_{n-1} \right]$$

$$V = 20 \left[\frac{40 + 28.5}{2} + 112.5 + 96 + 80.5 + 52.5 \right]$$

$$V = 20 [375.75]$$

$$V = 7515 \text{ m}^3$$

By trapezoidal method

By Simpson's Rule $\Rightarrow V = \frac{d}{3} [A_1 + A_n + 4(A_2 + A_4 + \dots + A_{n-1}) + 2(A_3 + A_5 + \dots + A_{n-2})]$

$$V = \frac{d}{3} [40 + \overset{28.5}{112.5} + 4(112.5 + 80.5) + 2(96 + 52.5)]$$

$$V = \frac{20}{3} [1137.5]$$

$$V = 7583.33 \text{ m}^3$$

Question) In a Proposed Reservoir the area Contouring within the Contours are

Contour (m)	100	95	90	85	80	75	70	65
Area	32	26 26	24	18	15	13	7	2

using Method of End area Calculate.

(i) Capacity of Reservoir when it is full at 100m level.

(ii) Elevation of water level when Reservoir is 60% full ignore the volume below 65m ~~below~~.

Answers) $V = d \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \dots + A_{n-1} \right]$

~~$$V = 5 \left[\frac{32 + 2}{2} + 26 + 24 + 18 + 15 + 13 + 7 \right]$$~~

$$V = 5 \left[\frac{32 + 2}{2} + 26 + 24 + 18 + 15 + 13 + 7 \right]$$

$$V = 600 \text{ m}^3$$

(ii) when Reservoir is 60% full then volume, $V = 0.6 \times 600 = 360 \text{ m}^3$

$$\text{for volume } 95\% = d \left[\frac{A_2 + A_8}{2} + A_3 + A_4 + \dots + A_7 \right]$$

$$V_{95} = 5 \left[\frac{26 + 2}{2} + 24 + 18 + 15 + 13 + 7 \right] = 455 \text{ m}^3, h = 95 \text{ m}$$

$$V_{90} = 5 \left[\frac{24 + 2}{2} + 18 + 15 + 13 + 7 \right] = 330 \text{ m}^3, h = 90$$

Now 360 m^3 lies between these two by Interpolating
for $V = 360 \text{ m}^3$, $h = 91.2 \text{ m}$

CONTOURING

- The Relative Position of different Points on the surface of Earth in Plan are Represented by a map in order to Enhance the significance of these maps. Relative Elevation of these Points is also Indicated with the help of Contour.
- Contours are defined as Imaginary line Passing through the Points of Equal Elevation over the surface of Earth.

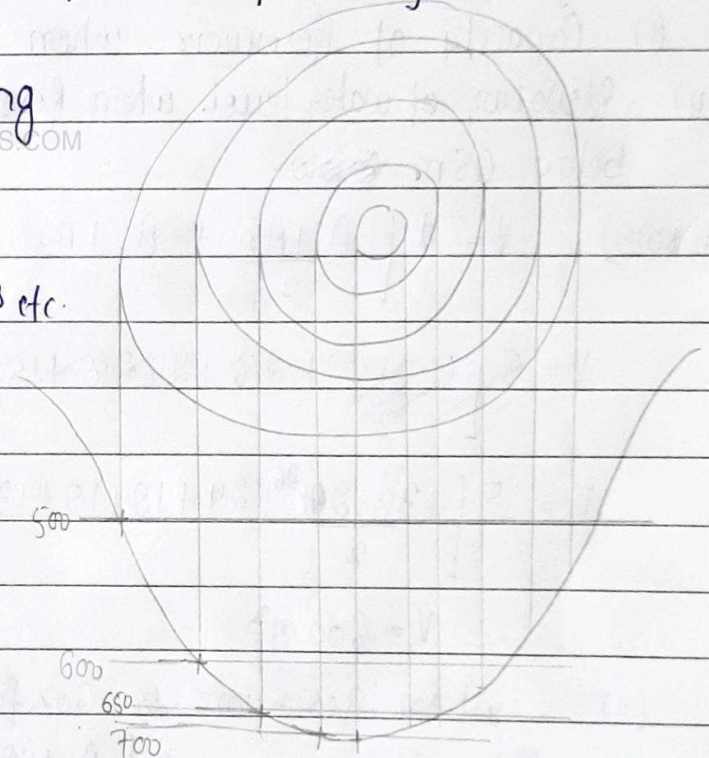
OR

- It may also be defined as Intersection of level surface with ground surface.

Note: If Contour are drawn below the water surface these are termed as submarine Contour/ fathoms/ Bathymetric Curve.

Contours are used in Engineering work in many ways.

- (1) Precise location of Roads, Canals, Railways tracks, sewers, Pipelines etc.
- (2) Planning and designing of dam, Reservoir, aqueducts etc.
- (3) ~~Reservoir~~ selection of sites of new Industry.
- (4) Deciding the Profile of the Country.
- (5) To Estimate the Quantity of Earthwork in Cutting and filling.



CONTOUR INTERVAL

- It is the Vertical distance between two consecutive Contour.
- It is desirable to have Constant Contour Interval, However in some special case it become difficult to make an

an Idea about steepness or flatness of slope.

- It is generally kept in range of 1-15m
- Smaller is the Contour Interval, more is the precision of accuracy.

- The Contour Interval depends upon several factors

(a) Scale of Map : Inversely Proportional.

scale is small then Contour Interval should be large.

(b) The Purpose and Extent of Survey: For the detailed design, small Contour is used and for large survey large Contour Interval is used.

(c) Nature of ground: For, Flat $\rightarrow$ Contour Interval less
steep $\rightarrow$ Contour Interval more

(d) Time and Cost: Time less then Contour Interval large or vice versa.
Cost less then greater Contour Interval & vice versa.

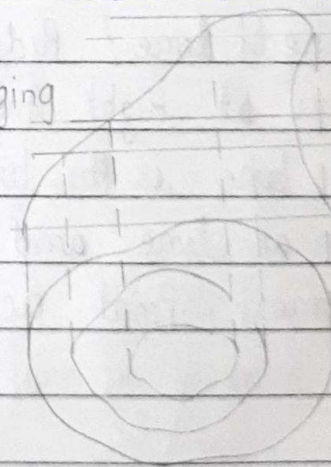
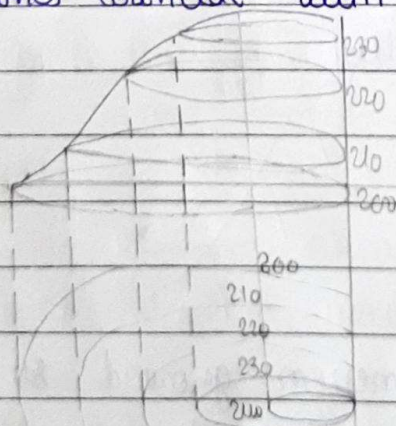
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PROPERTIES OF CONTOUR LINES

- The Elevation of Contours is being Marked on Contour itself
[Hence zero Contour ~~inside~~ Indicate Coast line, MSL]
- Two, Contour line do not intersect each other except in the case of overhanging cliff or Cave in Hill side.

- In Case of Vertical cliff
(generally found near sea coast)
the Contour lines coincide with each other.

overhanging
cliff or Cave



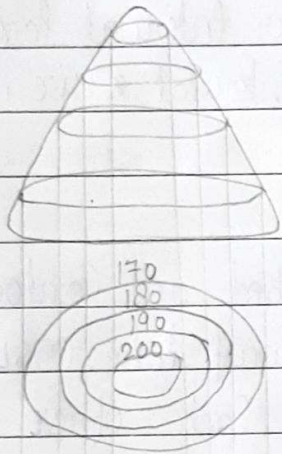
- A Contour line a closed Curve However it may close inside or outside the plan depending upon topography
- Equally spaced Contour signifies uniform slope (same slope), However magnitude slope will govern the magnitude of the spacing of the Contour

steep uniform

flat uniform

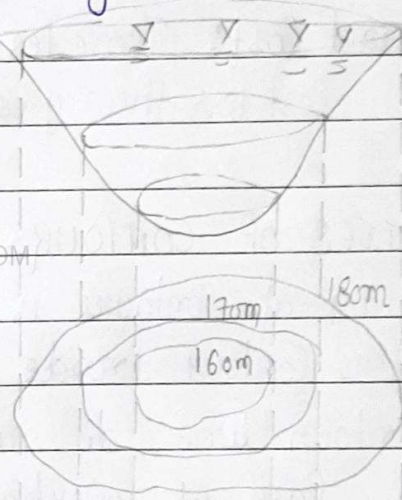
non-uniform

- A set of closed Contour Having Higher elevation (values) inside and lower values outside Represent hilltop or cliff and lower elevation inside and higher elevation outside Represent depression, river, lake, Ravine, Valley etc.

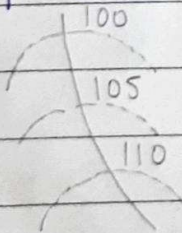
Hill
Rock

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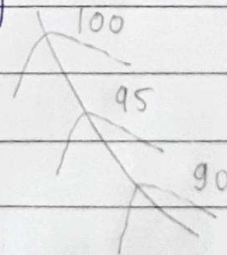
VALLEY



- Watershed line / Ridge line and Valley line / THAWLED, cross Contours at Right Angles
- Ridge line is the line joining the top most point Across the diff section of line and Valley line is the line joining the lowermost Point across a Valley

Ridge
line

VALLEY LINE



- Irregular Contour Represent uneven ground surface
- The direction of the steepest ~~slope~~ slope is along the shortest distance Perpendicular between the Contour.

PROPERTY OF LAND FEACHER TYPICALLY FOUND ON EARTH SURFACE.

(A) HIGH LYING FORMS: Ex hillrock, plateau These Contours are more or less Circular in shape with Elevation Increasing inwards direction.

(B) Slope SCRAP: These Contour are closely spaced and over-crowded

Note: A Very steep slope is termed as scrap and a high scrap is termed as crag.

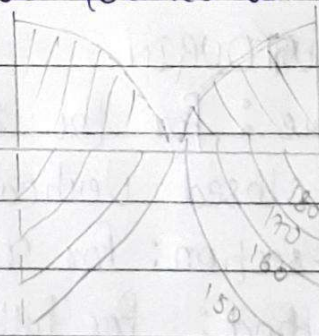
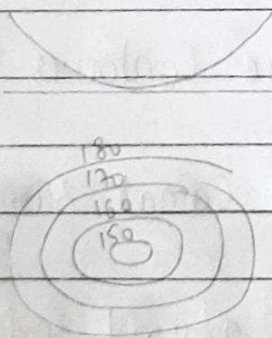
(C) Low LYING FORMS: Ex. Valley, Ravines, lakes/Pond etc. These Contours would be having Higher Elevation outward.

(i) Ravine: It is the depression on the Earth surface elongated on one side with bottom inclined towards one side.

(ii) Valley: It is broad Ravine with gentle sloping bottom.

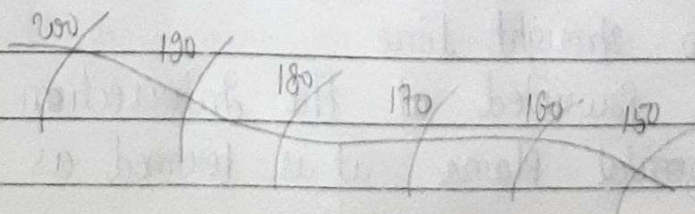
(iii) Depression: If the ground is low as compared to the surrounding land and side slopes are gentle it is termed as depression

(iv) GORGE / CANYONS: If the Valley floor is narrow and steep side slopes are available it is termed as gorge



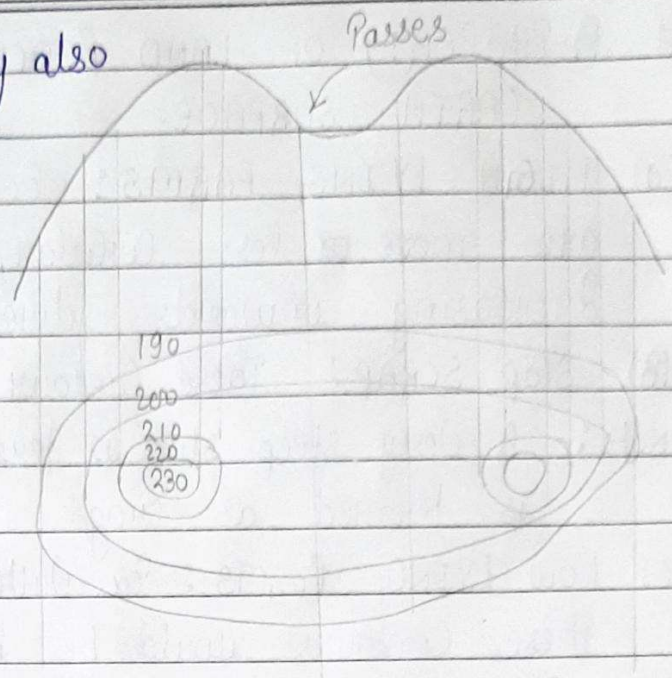
(v) Saddle: The lowest point on watershed line is termed as Passages.

Passages is low lying area enclosed passing through high lands forms [hill rock/mountains] on both side

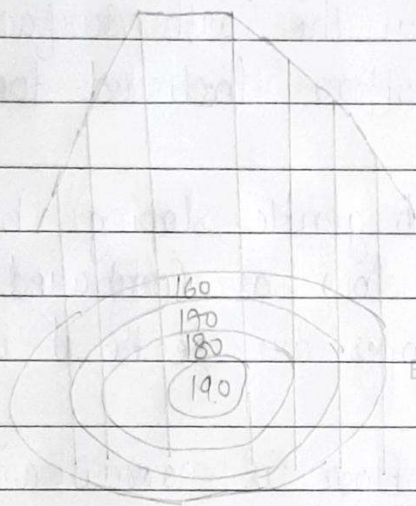


Isometric section

Through these Passes River stream may also
 Pass
 when the low lying area is broad
 it is termed as saddle



(vi) Escarpment: It is a
 high land having narrow
 top flat with steep slope on
 outside (scarp) and gentle
 slope of an other side



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 Escarpment

Method of CONTOURING

- Method of Square: For the small ^{open} area where Contours are Required at a closed Vertical Interval.
- Method of cross-section: For road, railway and Canal survey.
- Tacheometry method: For hilly area.
- DIRECT METHOD: For large scale maps when ground surface is Irregular.

CURVES

These Curves are the transition which are provided at the Intersection of two straight line

- If Curves are provided at the Intersection of two straight line in horizontal plane it is termed as horizontal Curve.

And if it is provided at the intersection of two straight line in vertical plane it is termed as Vertical Curve

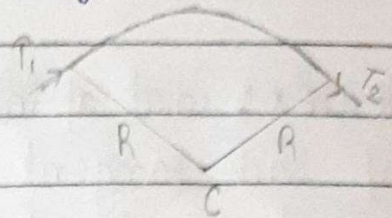
- Horizontal Curve provided are generally Circular and Vertical Curves provided are generally Parabolic.

- Horizontal Curves are generally of three types.

(a) Simple Circular Curve:

It consists of an arc of circle

- The curve is tangential to 2 straight line of the path / Route.

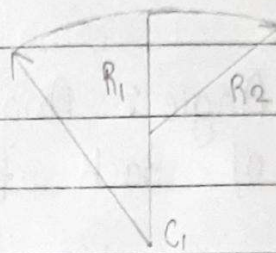


(b) Compound Curve

It consists of two arcs of circle of different radius and center, both

these arcs of circle are tangential to a common line

- The center of curvature of these two arcs lies on the same side of the common tangent.

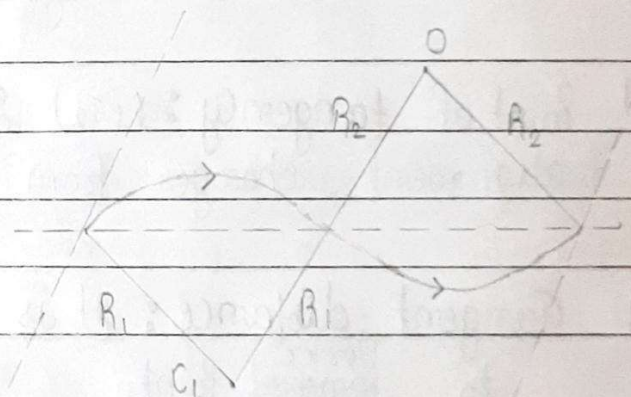


(c) Reverse Curve

These are provided in between the two straight lines of a route which are parallel to each other or having a small angle between them.

- It also consists of two circular arcs of same or different radius and center on opposite sides of a common tangent.

Note: Compound Curve and Reverse Curve are provided for low speed on Road and Railway.



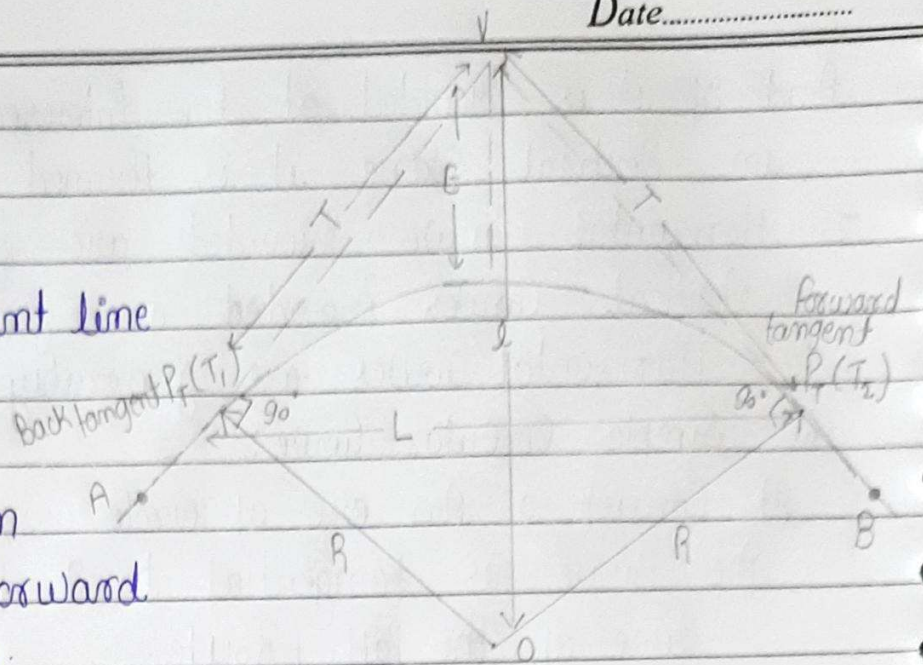
COMPONENT OF HORIZONTAL CURVE

BACK TANGENT (AT₁) Tangent line before the beginning of curve (AT₁)

(T_1) PT $\rightarrow$ Tangent Curve
 PT (T_2) $\rightarrow$ Curve Tangent

(b) Forward tangent: Tangent line at the end of the curve.

(c) VERTEX: Point of Intersection of back tangent and Forward tangent is called Vertex.



(d) Deflection Angle: Angle between the line formed by the Extension of Back tangent and Forward tangent.

(e) Point of Curvature: It is the beginning of the Curve where the alignment changes from a tangent to Curve also known as Tangent Curve.

(f) Point of tangency: (PT_2) It is the end of the Curve where the alignment changes from Curve to tangent.

(g) Tangent distance: It is the distance between Point of Curvature to Vertex Point.

(h) External distance: or Apex distance: It is the distance from mid point of Curve to VPI.

(i) mid ordinate: It is the ordinate from mid point of the long chord to mid point of Curve.

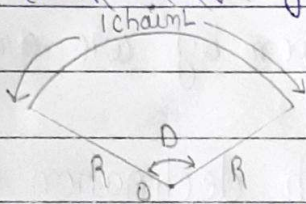
(j) Length of Curve: It is total length of Curve from T_1 to T_2 .

(k) Right hand Curve: It is the Curve deflects to the Right of the direction of the Progress of survey, it is called Right hand Curve.

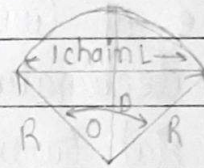
(ii) Left Hand Curve: It is the Curve deflects to the left of the direction of Progress of the survey it is called LHC.

~~DESIGNATION~~ DESIGNATION / DESCRIPTION OF CURVE

- A Curve Can be designated or described by its radius R or degree of the Curve
- Degree of Curve signifies the Angle subtended by the Curve at its Center
- Here Curve signifies either its arc or chord.
- According to arc definition Degree of Curve is the angle subtended at the Center by an arc of one chain length [20m/30m] it is generally used in Highway Practice.
- According to chord definition, degree of Curve is Angle subtended at the Center by the chord



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As Per arc definition

if 1 chain length = 30m,

$RD = \text{length of arc} = 1 \text{ chain length}$

$$D = \frac{\text{arc}}{R} = \frac{2\pi R \times}{180}$$

$$RD \times \frac{\pi}{180} = 30$$

$$R = \frac{1718.0}{D} \approx \frac{1720}{D}$$

if chain length = 20m

$$\frac{RD \times \pi}{180} = 20 \Rightarrow R = \frac{1145.91}{D}$$

$$R = \frac{1146}{D}$$

$$\left[\begin{aligned} \frac{2\pi R}{360} &= \frac{20}{D} \\ R &= \frac{1146}{D} \end{aligned} \right]$$

As Per chord definition

$$\sin \frac{D}{2} = \frac{1 \text{ chain length}}{2R}$$

$$R = \frac{L}{(\sin D/2)}$$

$$R = \frac{\text{1 chain length}}{(2 \sin D/2)}$$

if 1 chain = 30m

$$R = \frac{30}{2 \sin D/2} = \frac{30}{2 \times \frac{D}{2} \times \pi/180} = \frac{1718.87}{D}$$

if 1 chain length = 20m

$$R = \frac{1146}{D}$$

Note: - Degree of Curve is Inversely Proportional to Radius to Curve.

- Hence for sharp Curve [for which radius is less] degree of Curve is large and for Flat Curve [for which radius is more] degree of Curve is less.
- If degree of Curve is small, Result given by arc and chord declination is same.
- Hence for Flat Curve Results obtained by both declination is same but for sharp Curve it is difficult.

ELEMENTS OF A SIMPLE CIRCULAR CURVE

(i) Length of Curve (L)

$$\text{Length of Curve } T_1CT_2 (L) = \frac{\pi R \cdot \Delta}{180}$$

(ii) Tangent length (T) In ΔVT_1O , $\tan \frac{\Delta}{2} = \frac{T}{R} \Rightarrow T = R \tan \frac{\Delta}{2}$

(iii) Long chord length (L)

$$\text{In } \Delta T_1DO, \sin \frac{\Delta}{2} = \frac{T_1D}{R} \Rightarrow T_1D = R \sin \frac{\Delta}{2}$$

$$\text{Length of chord } T_1T_2; L = 2R \sin \frac{\Delta}{2}$$

(iv) External distance (E)

$$\text{In } \triangle VTO, \frac{\cos \frac{\Delta}{2}}{2} = \frac{R}{VO} \Rightarrow VO = \frac{R \cos \frac{\Delta}{2}}{2} = R \sec \frac{\Delta}{2}$$

$$\text{External distance } VC = E = VO - OC$$

$$E = VO - R = \frac{R \sec \frac{\Delta}{2}}{2} - R$$

$$E = R \left(\frac{\sec \frac{\Delta}{2}}{2} - 1 \right)$$

(v) Mid ordinate (m)

$$\text{In } \triangle DTO, \frac{\cos \frac{\Delta}{2}}{2} = \frac{OD}{R} \Rightarrow OD = \frac{R \cos \frac{\Delta}{2}}{2}$$

$$\text{mid ordinate } CD = m = OC - OD$$

$$m = R - \frac{R \cos \frac{\Delta}{2}}{2} = R \left[1 - \frac{\cos \frac{\Delta}{2}}{2} \right]$$

(vi) CHAINAGE OF TANGENT POINT.

- In General chainage of Point of Intersection V is known

chainage of tangent Point T_1 = chainage of V - T

chainage of tangent Point T_2 = chainage of T_1 + L

Note: chainage of tangent Point $T_2 \neq$ chainage of V + T. As the Projection of Curve is from T_1 to T_2 via "C" not V

Question) Two straight line Intersect At chainage of 1000.5 and angle of Intersection is 60° if the Radius of Curve is 500m, Compute All the Elements of the Curve

$$\text{Ans) } L = \frac{\pi R \cdot \Delta}{180} = \frac{\pi \times 500 \times 60^\circ}{180} = 523.6$$

$$T = R \tan \frac{\Delta}{2} = 500 \tan 30^\circ = 288.67 \text{ m}$$

$$3) \text{ Long chord length } l = \frac{2R \sin \frac{\Delta}{2}}{2} = 2 \times 500 \sin 30^\circ = 500$$

$$4) E = R(\sec 30^\circ - 1) = 500(\sec 30^\circ - 1) \Rightarrow 77.35$$

$$5) m = 500[1 - \cos 30^\circ] = 66.98 \text{ m}$$

$$6) \text{Chainage of tangent Point } T_1 = \text{chainage of } V - T_1$$

$$= -289 + 1000.5 = 711.5 \text{ m}$$

$$7) \text{Chainage of tangent Point } T_2 = \text{chainage of } T_1 + l$$

$$= 711.5 + 523.5 = 1235.0 \text{ m}$$

SETTING OUT OF CURVES: It is the process of locating the points along the length of curve either at equal or convenient distance.

- setting of curve can be broadly done by two method.

(1) LINEAR METHOD: These are the methods in which setting is done by measuring distance.

(2) Angular Method: These are the methods in which setting is done by distance as well as angle.

- setting of curve can be broadly ~~done by two method~~ classified on the basis of instrument use.

(A) TAPE METHOD (B) TAPE AND THEODOLITE

(C) TWO THEODOLITE METHOD (D) TACHEOMETER (E) TOTAL STATION METHOD.

(A) LINEAR OFF-SET METHOD

$$CD = OC - OD, \quad OC = R$$

$$OD = \sqrt{R^2 - (L/2)^2}$$

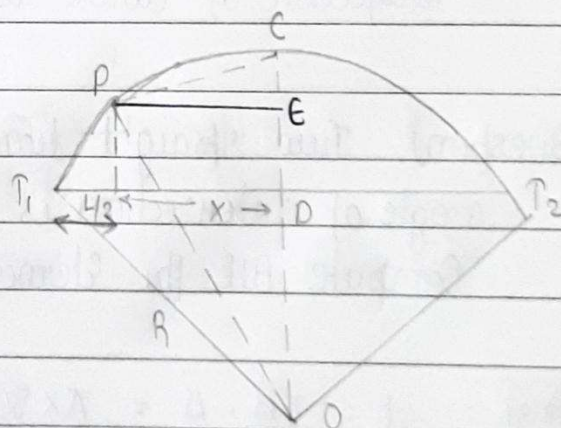
$$CD = R - \sqrt{R^2 - (L/2)^2}$$

$$\text{In } \triangle PEO, \quad R^2 = x^2 + (OD + y)^2$$

$$(OD + y)^2 = R^2 - x^2$$

$$y = \sqrt{R^2 - x^2} - OD$$

$$\Rightarrow y = \sqrt{R^2 - x^2} - \sqrt{R^2 - (L/2)^2}$$



(2) OFFSET FROM TANGENT

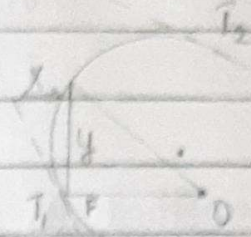
$$X = \tau_1 f = \tau_{1,0} - f_0$$

$$\tau_{1,0} = R, f_0 = \sqrt{R^2 - y^2}$$

$$X = R - \sqrt{R^2 - y^2}$$

$$X = R - \left[R^2 - \frac{1}{2} y^2 + \dots \right]$$

$$X = \frac{y^2}{2R}$$



(3) OFFSET FROM CHORD PRODUCE

$$O_1 = C_1 \delta_1 \Rightarrow \delta_1 = \frac{O_1}{C_1} \quad (i)$$

$$C_1 = R \times 2\delta_1 \Rightarrow \delta_1 = \frac{C_1}{2R} \quad (ii)$$

(i) & (ii)

$$\frac{O_1}{C_1} = \frac{C_1}{2R} \Rightarrow O_1 = \frac{C_1^2}{2R}$$

$$O_2 = O_1' + O_2''$$

$$O_2 = C_2 \delta_1 = \frac{C_2 C_1}{2R}$$

$$O_2'' = C_2 \delta_2 \Rightarrow \delta_2 = \frac{O_2''}{C_2}$$

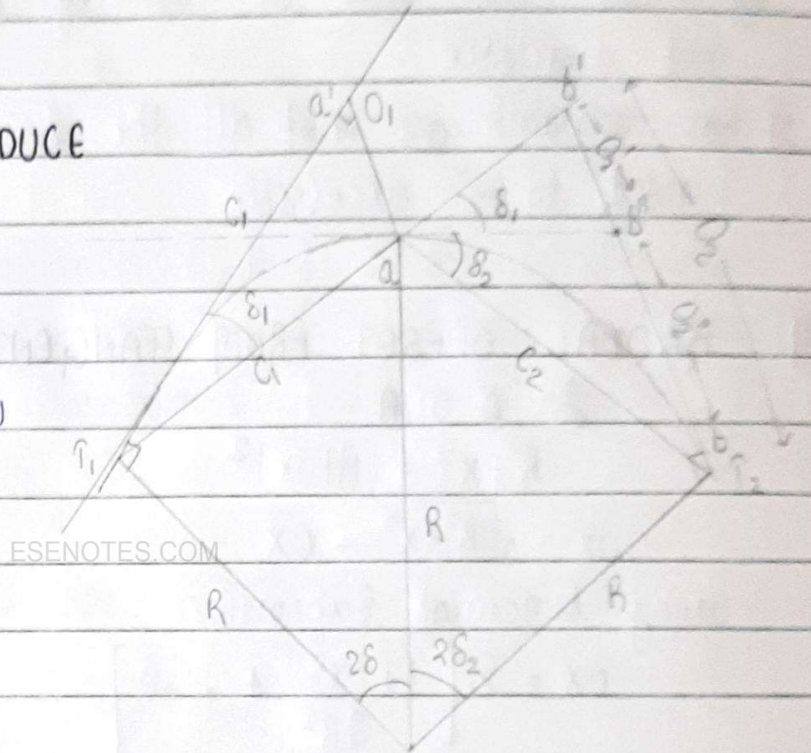
$$\text{Now, } C_2 = R \times 2\delta_2$$

$$\delta_2 = \frac{C_2}{2R}$$

$$O_2 = \frac{C_2 (C_1 + C_2)}{2R}$$

$$\text{Similarly } O_3 = \frac{C_3 (C_2 + C_3)}{2R}$$

$$O_m = \frac{C_m (C_{m-1} + C_m)}{2R}$$



(2) OFFSET FROM TANGENT

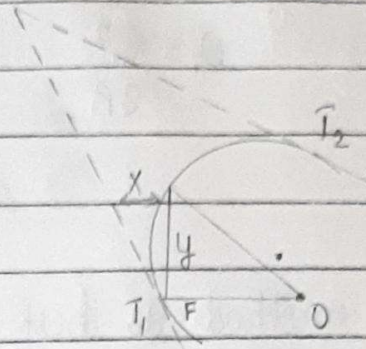
$$X = \tau_1 F = \tau_{1,0} - F_0$$

$$\tau_{1,0} = R, F_0 = \sqrt{R^2 - y^2}$$

$$X = R - \sqrt{R^2 - y^2}$$

$$X = R - \left[R^2 - \frac{1}{2} y^2 + \dots \right]$$

$$X = \frac{y^2}{2R}$$



(3) OFFSET FROM CHORD PRODUCE

$$O_1 = C_1 \delta_1 \Rightarrow \delta_1 = \frac{O_1}{C_1} \quad \text{--- (i)}$$

$$C_1 = R \times 2\delta_1 \Rightarrow \delta_1 = \frac{C_1}{2R} \quad \text{--- (ii)}$$

(i) & (ii)

$$\frac{O_1}{C_1} = \frac{C_1}{2R} \Rightarrow O_1 = \frac{C_1^2}{2R}$$

$$O_2 = O_2' + O_2''$$

$$O_2 = C_2 \delta_1 = \frac{C_2 C_1}{2R}$$

$$O_2'' = C_2 \delta_2 \Rightarrow \delta_2 = \frac{O_2''}{C_2}$$

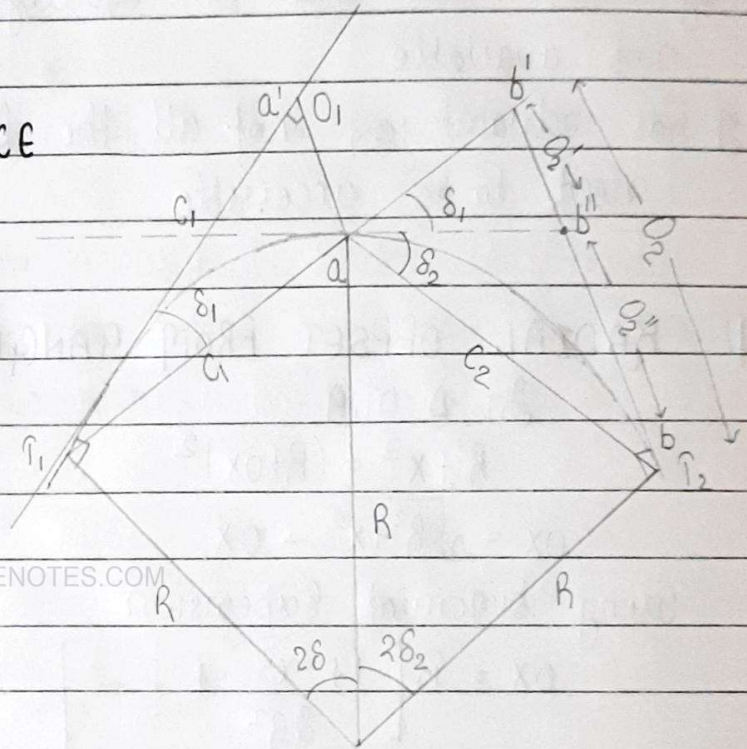
$$\text{Now, } C_2 = R \times 2\delta_2 \quad \rightarrow \quad \frac{O_2''}{C_2} = \frac{C_2}{2R} \Rightarrow O_2'' = \frac{C_2^2}{2R}$$

$$\delta_2 = \frac{C_2}{2R}$$

$$O_2 = \frac{C_2(C_1 + C_2)}{2R}$$

$$\text{Similarly } O_3 = \frac{C_3(C_2 + C_3)}{2R}$$

$$O_n = \frac{C_n(C_{n-1} + C_n)}{2R}$$



Practically $C_2 = C_3$ $C_{n-1} = 1$ chain length

$$O_1 = \frac{C_1^2}{2R} \quad O_2 = \frac{C_1 C + C^2}{2R}$$

$$O_3 = O_4 = O_5 = \dots O_{n-1} = \frac{C^2}{R}$$

Note: This method is best method for longer curves of large Radius by linear mtd and is usually used when theodolite is not available.

- It has advantage, not all the land b/w tangent and curve need to be accessible

(4) RADIAL OFFSET FROM TANGENT

In ΔOT_1A

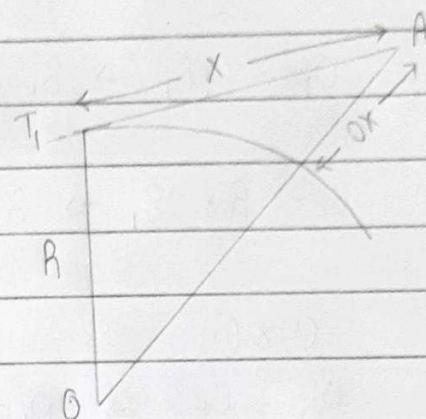
$$R^2 + x^2 = (R + ox)^2$$

$$ox = \sqrt{R^2 + x^2} - R$$

using binomial Expansion

$$ox = R \left[1 + \frac{x^2}{2R^2} + \dots \right]$$

$$ox = \frac{R^0 + x^2}{2R} - R \Rightarrow \frac{x^2}{2R}$$

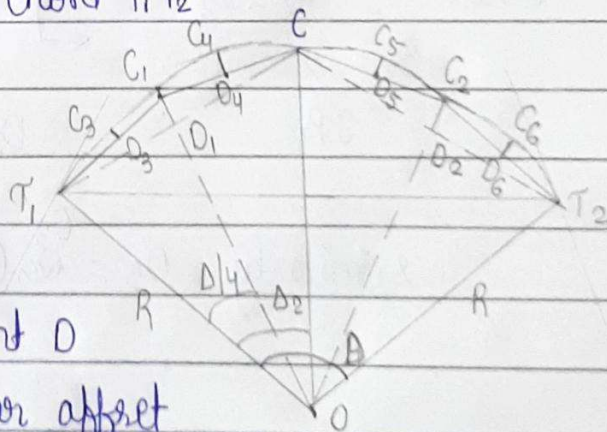


(5) SUCCESSIVE BI-SECTION OF CHORD OR ARC

Let T_1 and T_2 be the tangent Point chainage of which is known
- Point C is established by bisecting the chord T_1T_2 at Point D and Marking Perpendicular offset CD at Point D Equal to mid-ordinate $[R(1 - \cos \Delta/2)]$

- Join T_1C and T_2C
- Now T_1C and T_2C are bisected At Point D and D_2 and Marking the Perpendicular offset

D_1C_1 and D_2C_2 Equal to $R[1 - \cos \Delta/2]$. Hence Point C_1 and C_2 are obtained on Curve



Now join T_1C_1 , C_1C , C_2C , C_2T_2 and carryout the successive bi-section of these chord to locate the point on chord to locate the point on curve.

(B) ANGULAR METHOD

(i) TAPE AND THEODOLITE METHOD: In this method tape is used for making linear measurement and theodolite is used for making angular measurement.

It is also termed as tangential deflection angle measurement and Rankine method.

- For this method it is recommended that chord length does not exceeds $R/20$ As this insures that the length of chord may be approximated to the corresponding length of arc.

$$\delta_1 = \Delta_1$$

$$\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$$

$$\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$$

Here $\delta_1, \delta_2, \dots, \delta_n$ are the tangential angle made by successive curves T_1A, T_1b, T_1C with back tangent at T_1 .

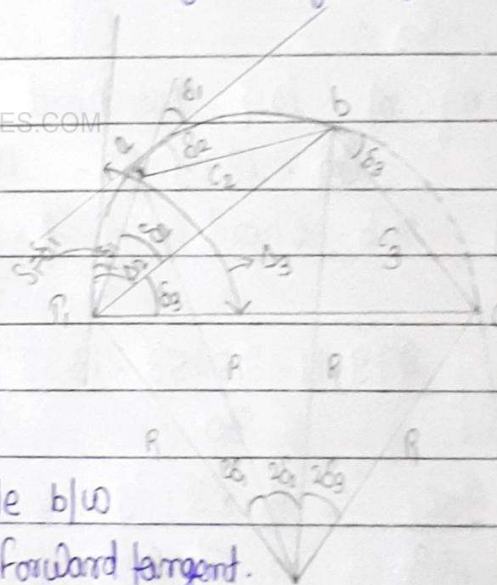
Note: Here $\Delta_n = \frac{1}{2}$ main deflection angle b/w back tangent and forward tangent.

$$\Delta_n = \frac{\Delta}{2}$$

$$R_2 S_1 = C$$

$$S_1 = \frac{2\pi R \times 2\delta_2}{360}$$

$$S_1 = \frac{90C_1}{\pi R}, \quad S_2 = \frac{90C_2}{\pi R}, \quad \dots, \quad S_n = \frac{90C_n}{\pi R}$$



Date.....

Question) Two tangent intersect at chainage of 1200m the deflection angle being 40° Compute the data required to set out simple Circular Curve of Radius 400m, using deflection Angle method
Make 30m chord length = 1 chain length

Ans) $T_L = \frac{R \tan \frac{\Delta}{2}}{2} = \frac{400 \tan 20^\circ}{2} = 72.79 \times 2 = 145.58$

$Cl = \frac{\pi R \times \Delta}{180} = \frac{3.14 \times 400 \times 40}{180} = 279.11$

Ch. of $T_1 = 1200 - 145.58 = 1054.42$

Ch of $T_2 = 1054.42 + 279.11 = 1333.53$

Chainage of 1st subchord = 1080m

$C_1 = 1080 - 1054.42 = 25.58m$

$C_2 = C_3 = \dots = C_{n-1} = 30m$

Chainage of last subchord

$C_n = 1333.53 - 1300 = 33.53m$

No of sub-chord = $\left[\frac{L - C_1 - C_n + 2}{30} \right]$

$= \left[\frac{279.11 - 25.58 - 33.53}{30} \right] + 2 = 10$

$\delta_1 = \frac{90 C_1}{\pi R} = \frac{90 \times 25.58}{\pi \times 400} = 1.832$

$\Delta \alpha = \Delta \alpha_{n-1} + \delta_n$

Subchord length C (m)

Tangential (δ), $\delta = 90C/\pi R$

Deflection Angle (α)

$C_1 = 25.58$

$\delta_1 = 1.832$

$\Delta \alpha = 1.832$

$C_2 = 30$

$\delta = 2.148$

$= 3.98$

C_3

$\delta = 2.148$

6.128

C_4

8.276

C_5

10.424

C_6

12.527

C_7

14.72

C_8

16.868

C_9

19.016

C_{10}

$\delta_{10} = 0.96$

19.986

Note: The above procedure of tape and theodolite method as discussed is for Right hand Curve but if the Curve is left hand Curve, the theodolite is set up at Point of Tangency (T_2) and Vernier is set to be zero & line of sight is directed toward V.

For the first Peg, the Vernier A is set at an angle of $\Delta_1 = 360^\circ - \Delta_1$.

(b) TWO THEODOLITE METHOD.

$$\Delta_1 = \delta_1$$

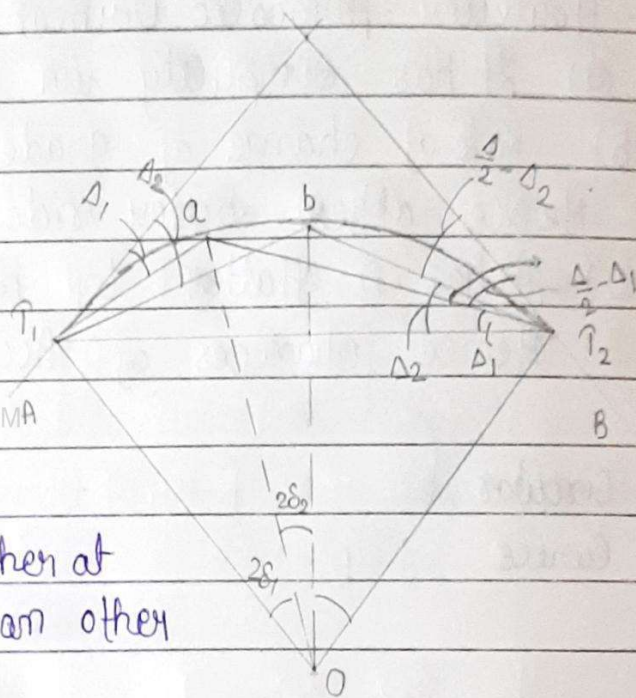
$$\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$$

$$\Delta_n = \Delta_{n-1} + \delta_n$$

In this method 2 theodolite are used for Angular measurement

No linear measurement is being carried out

one of theodolite is placed at T_1 and other at T_2 . This method is quite Accurate than other method



- It is suitable to be used when taping and chaining is difficult i.e. where ground is rough.
- In this method the error in setting out is not carried forward at each point is fixed independently.
- This method is based on property of circle that the chord makes an angle at the center of circle which is twice the angle made by chord at any other point on the circle. Hence it can be concluded that the angle made by the chord with the tangent at a point is equal to the angle which chord subtended at any other point on the circle.

(3) TACHEOMETER METHOD: In this case both linear and Angular measurement is done by tacheometer.

- Here process is similar to tape and theodolite method.

VERTICAL CURVES

The Curves use to connect Lines of different slopes / Grade of Railways and highways & termed as Vertical Curve.

- This Curve may either be Circular or Parabolic

- However Parabolic Vertical Curve are preferred as

(a) It has Simplicity in lying and Computation work

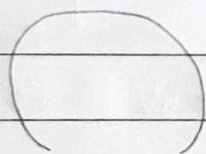
(b) Rate of change of grade is uniform throughout this Curve

Hence offer better ride Quality

(c) It has A flatter top hence offers large sight distance

Hence chances of Accident Reduces

Circular Curve



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Parabolic Curve.



The General Eqⁿ for Parabolic Curve

$$y = ax^2 + bx + c$$

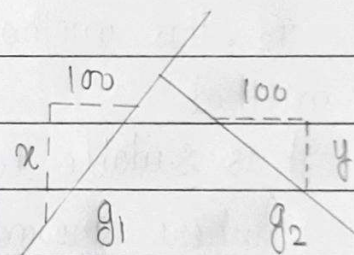
$$\frac{dy}{dx} = 2ax + b$$

at

$$\text{at } x=0 \quad \frac{dy}{dx} = g_1$$

$$b = g_1$$

$\frac{d^2y}{dx^2} = 2a \Rightarrow$ Rate of change of gradient is uniform which offers Best riding Quality



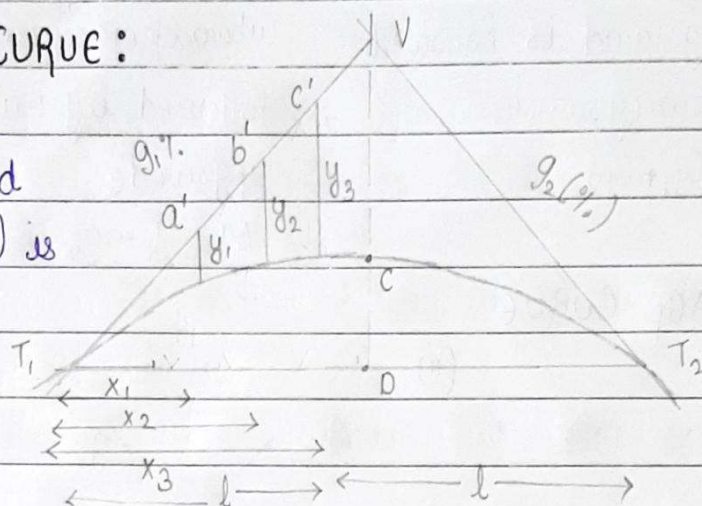
$y \propto x^2$
 $y = Kx^2$ [x' = distance from tangent point T_1 along the tangent x']

Since Parabolic Curve is flat at top Normal offset y' is Equal to Vertical offset $y' = y$

SETTING OF VERTICAL CURVE:

Length of Curve, chainage and RL of Point of Intersection (V) is known

- RL of $T_1 = \text{RL of } V - g_1 l$
- RL of $T_2 = \text{RL of } V - g_2 l$
- RL of D = $\frac{1}{2} [\text{RL of } T_1 + \text{RL of } T_2]$



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$$VD = \text{RL of } V_c - \text{RL of } D$$

$$V_c = \frac{1}{2} VD$$

$$y_1 = \frac{C}{2l} x_1^2 \quad (1)$$

$$V_c = \frac{C}{2l} l^2 \quad (2)$$

$$\text{From (1) and (2)} \quad \frac{y_1}{V_c} = \frac{x_1^2}{l^2} \Rightarrow y_1 = \left[\frac{x_1}{l} \right]^2 V_c$$

$$y_2 = \left[\frac{x_2}{l} \right]^2 V_c \quad \dots \quad y_n = \left[\frac{x_n}{l} \right]^2 V_c$$

$$\text{RL of } a' = \text{RL of } T_1 + g_1 x_1$$

$$\text{RL of } b' = \text{RL of } T_1 + g_1 x_2 + \dots \quad \text{RL of } n^{\text{th}} = \text{RL of } T_1 + g_1 x_n$$

$$\text{Now RL of } a = \text{RL of } a' - y_1$$

$$\text{RL of } b = \text{RL of } b' - y_2$$

RL of diff Points of Curve Can be Computer

Note: In Case of Sag Curve the offsets are to be added to the RL of Point on tangent to get the RL of Points on the Curve.

ALTERNATE METHODS FOR SETTING OF VERTICAL CURVE

Since Vertical Curve provided is Parabolic, Equation of which is given by

$$\text{At } x=0, y=0 \quad y = Ax^2 + bx + c$$

$$0 + c \Rightarrow c = 0$$

$$\frac{dy}{dx} = 2ax + b$$

$$\text{At } x=0 \quad \frac{dy}{dx} = y_1$$

$$y_1 = 2a(0) + b \Rightarrow b = g_1$$

$$y = ax^2 + g_1x \quad \text{--- (A)}$$

$$PQ = PR - QR \Rightarrow g_1x - y$$

$$\text{From (A)} \quad g_1x - y = -ax^2$$

$$PQ = -ax^2 \quad \text{Let } -a = C$$

$$h = -ax^2$$

$$h = Cx^2$$

Let h i.e. difference between elevation of tangent and Vertical Curve is termed as tangent Correction which is found as follow.

$$- \quad AT_1V = VA$$

$$- \quad AT_2 = AB + BT_2$$

$$- \quad AB = g_1 \cdot n \text{ chords}$$

$$- \quad BT_2 = -g_2 \cdot n \text{ chords}$$

$$- \quad A + g = g_1 \cdot n \text{ chords} - g_2 \cdot n \text{ chords}$$

$$= n(g_1 - g_2) \text{ chords}$$

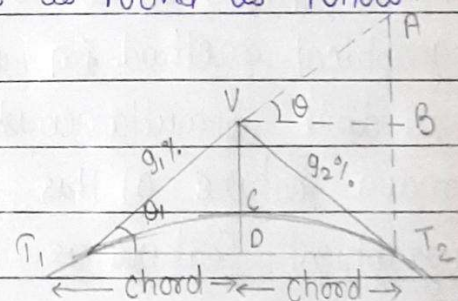
$$\text{Now tangent Correction } AT_2 = Cx^2$$

$$AT_2 =$$

$$(g_1 - g_2) \cdot n \text{ chords} = 4C(n \text{ chords})^2$$

$$C = \frac{g_1 - g_2}{4n \cdot \text{chords}}$$

$$4n \cdot \text{chords}$$



(A) ELEVATION BY TANGENT CORRECTION METHOD

$$RL \text{ of } T_1 = RL \text{ of } V - g_1 \cdot n \text{ chords}$$

$$RL \text{ of } T_2 = RL \text{ of } V - g_2 \cdot n \text{ chords}$$

Calculate the tangent Correction from $y = h = Cx^2$ for various

Various station to the Curve

$$y_1 = C_1^2 = C$$

$$h_2 = C(2)^2 = 4C$$

$$h_m = C(2m \text{ chords})^2 = 4(m \text{ chords})^2 C$$

RL of Corresponding Point (P) on tangent

$$RL \text{ of } P = RL \text{ of } T_1 + xg_1$$

- RL of Corresponding Point Q on Curve

$$RL \text{ of } Q = RL \text{ of } P \pm h \text{ [Tangent Correction]}$$

(B) Elevation by Chord Correction.

In this method diff in Elevation of successive Point on the Curve is determined for setting of Curve

- This diff is termed As chord Correction or chord gradient

PLANE TABLE SURVEY

- A Plane table is a surveying instrument used for surveying by graphical method in which field observation and plotting is done simultaneously

The main feature of this surveying is that chances of missing any topographical feature is very less, As it is carried out in field itself

- This is suitable for small and medium scale work mapping 1:10,000 to 1:25,000 where high accuracy is not ~~required~~ ^{desired}.

- Here, since plotting and observation are done simultaneously the time required is less

- No skilled man power or special instrument is required in this case, hence it is economical.

- Since plotting in this case done simultaneously checks can be applied in the field only.

- It does not require accurate result.

ELEMENTS OF PLANE TABLE SURVEYING

(i) **PLANE TABLE BOARD**: It is wooden drawing board made of well seasoned timber

Its upper surface kept smooth

- The table at the center can be attached to the tripod with the help of wing nut.

The boards are available in different sizes and are designated by

B_0 : 1500×1000 mm

B_1 : 1000×700 mm

B_2 : 700×500 mm

B_3 : 500×350 mm

(ii) **TRIPOD**: It is used for placing plane table board over it levelling of which is done with the help of tripod legs and levelling screws

(iii) **ALIDADE**: It is a straight edge ruler having an sighting device at its ends in the form of vanes. It is also used for drawing the lines

- It has 45cm long ruler [wooden or metallic] with graduations on one side which is bevelled [having slope] termed as fiducial edge.

- Vanes are being hinged on either side which are used for sighting the object.

(iv) **COMPASS**: Though compass is used for establishing the magnetic meridian

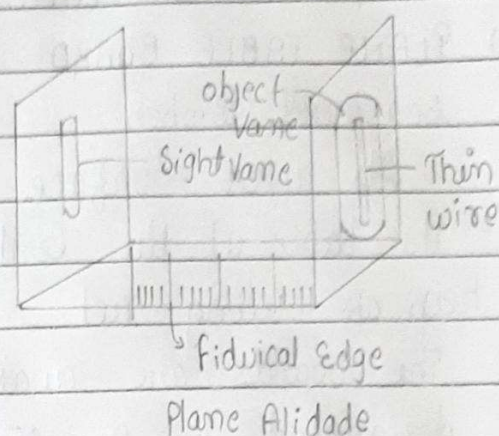
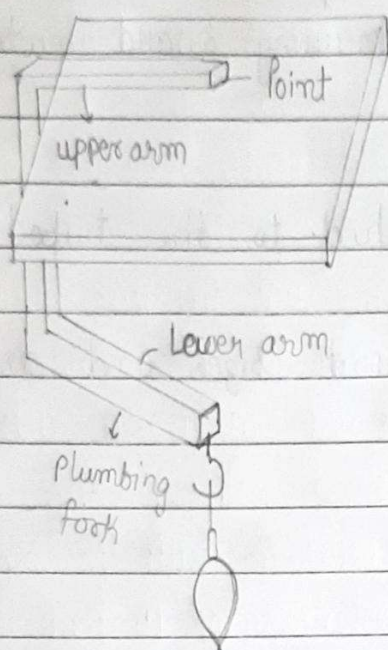
If needle provides an compass it will align itself along magnetic meridian, reading in compass corresponds to zero.

(v) **LEVEL TUBE**: Two level tubes at right angle to each other are placed over the plane table board. To set the table in horizontal plane.

(vi) **PLUMBING TORK**: It is a U-shaped, metallic form used to centering the table over the station

- It can also be used for transferring the points from ground over the table and vice-versa

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TEMPORARY ADJUSTMENT OF PLANE TABLE SURVEY.

- (i) The fiducial edge of alidade should be straight line.
- (ii) The two vanes should be perpendicular to the base of alidade.
- (iii) Centering: It is the process of bringing the plate point on table exactly over the ground station. It is done with the help of Plumbing fork.
- (iv) Levelling: It is the process of bringing plate table in horizontal plane. It is done with the help of tripod legs, levelling screw and checked with the help of level tube.

ORIENTATION: It is the process of keeping the plane table parallel to the position it occupied at first station.

It can be done by following method.

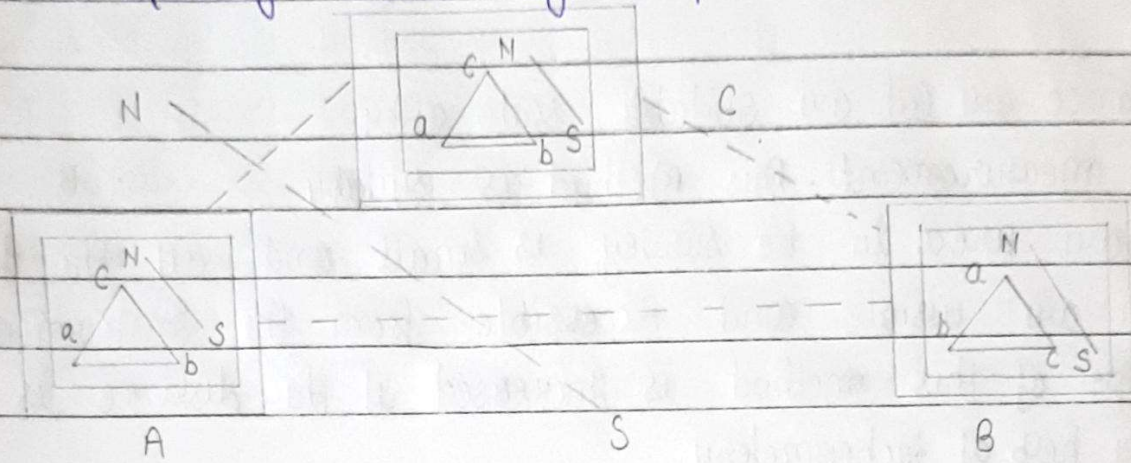
- (A) By Trough Compass: It is not suitable to be used in places which are affected by local compass attraction.

A Trough Compass is placed over the top right side corner of table in such a way the magnetic needle points towards exactly [N-S] [i.e. Reading on Compass must be zero].

— Draw a line along the edge of Compass

By Jaspal Sir

- Shift the table on next station. Place the Compass along N-S line
- Rotate the table till the magnetic needle coincides with N-S line or Reading shown by Compass is Zero

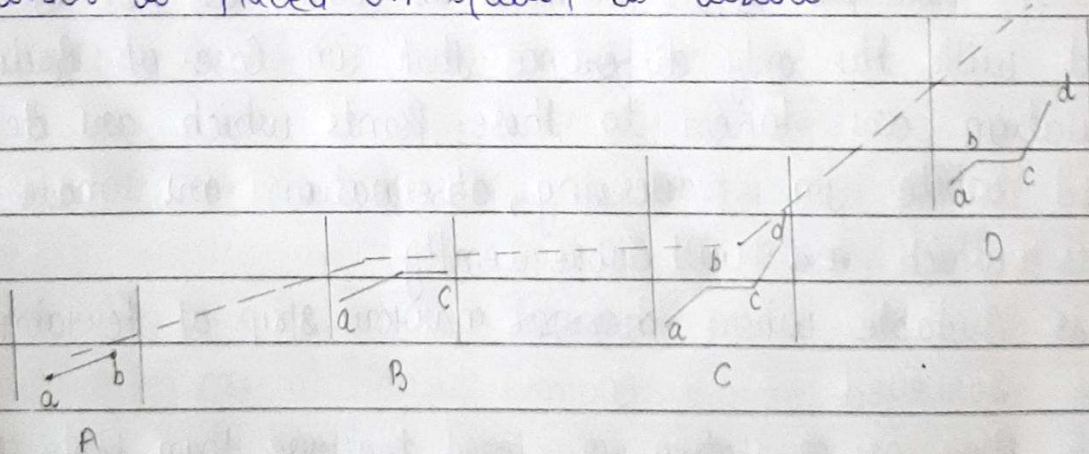


• This method of orientation is not accurate

- (2) By BACK-SIGHTING: This is more accurate method of orientation. In this method plane table is set on a new station and alidade is placed against the line chaining the new station with the previous station and the table is rotated until the line of sight bisect the previous station.

Process: Pivot the alidade at Point A, sight the Ranging Rod which is on B. make straight line along the fiducial edge of alidade

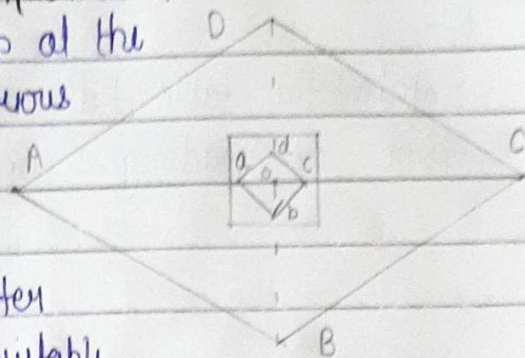
- Put alidade along the ba and Rotate the table till Ranging Rod which is placed on station is visible.



- (3) RESECTION: By Resection, Both orientation and fixing of plane table is achieved simultaneously.

METHOD OF PLANE TABLE SURVEY. (a) Radiation

In this method the instrument is set up at the station and rays are drawn to the various points or station which are to be plotted.



The distance are cut on suitable scale after

Actual measurement. This method is suitable

when an area to be surveyed is small and all the required stations are visible and accessible from the instrument station.

The scope of this method is increased if the distance is measured with the help of tachemetry.

(B) Traversing.

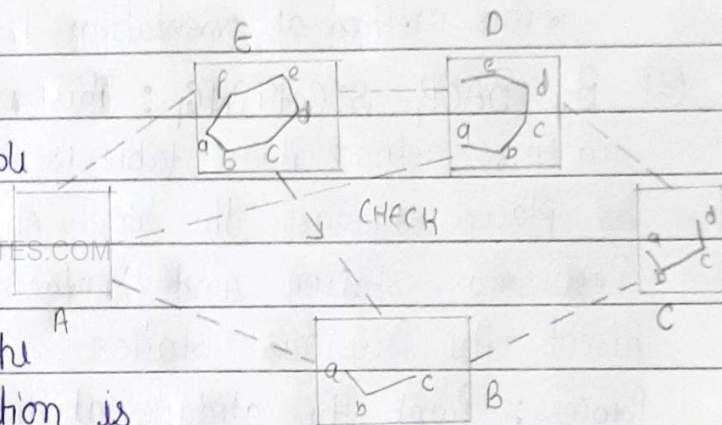
Plane table traverse has same principle as that of transit traverse.

- At each successive station table is

set, and foresight is taken at the

following station and its location is

plotted by measuring the distance with the help of chain tape or tachemeter b/w the two stations.



- Working principle of the method is same as that of Radiation method with the only difference that in case of Radiation the observation are taken to those points which are detailed or mapped while in traversing, observation are made of those points which ~~are~~ will subsequently.

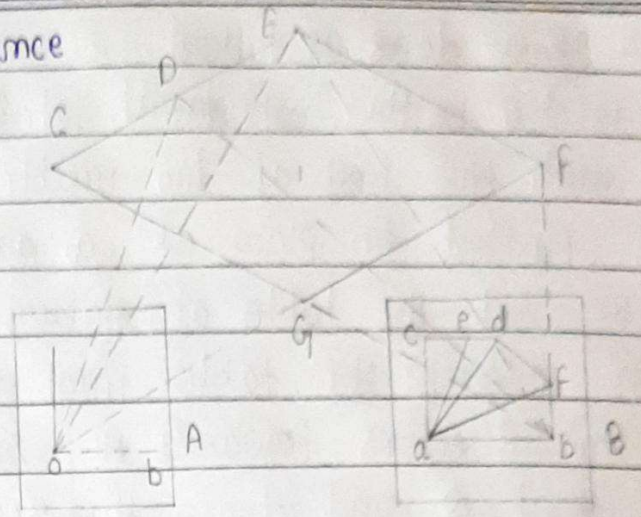
- It is suitable when ~~narrow~~ narrow strip of terrain is to be surveyed.

Note: If there are n stations in closed traverse then table is to be set on atleast $(n-1)$ station. To know the error of closure setting is to be done for atleast $(n-2)$ stations.

- Backsighting $\rightarrow (n-1)$

This method is used when the distance b/w Point and Instrument station is large or Cannot be measured Accurately

The location of object is determining by sighting at the object from two plane table station which are previously plotted



The line joining the two instrument station is termed as BASE LINE. In this method no linear measurement is required except that of baseline.

(d) RESECTION METHOD: Resection is the process of determining the plotted position of the station occupied by the plane table, by means of sight taken towards known points, location of which have been plotted.

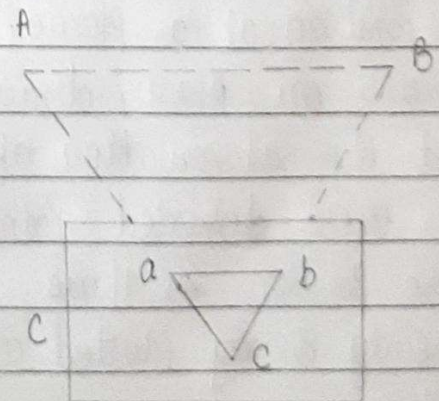
- These rays or line being drawn from plotted position to the locations of un-plotted position, are termed as Resectors.
- If the plane table is not properly oriented at the station to be ~~oriented~~ plotted the intersection of Rays/Lines from plotted points will not give the correct location of the instrument station.

The Problem of orientation is solved as follow

- (A) Resection after orientation by Compass
- (B) ee ee ee ee Back sighting
- (C) ee ee ee ee Two point method
- (D) ee ee ee ee Three Point Problem.

(1) Resection after orientation by Compass

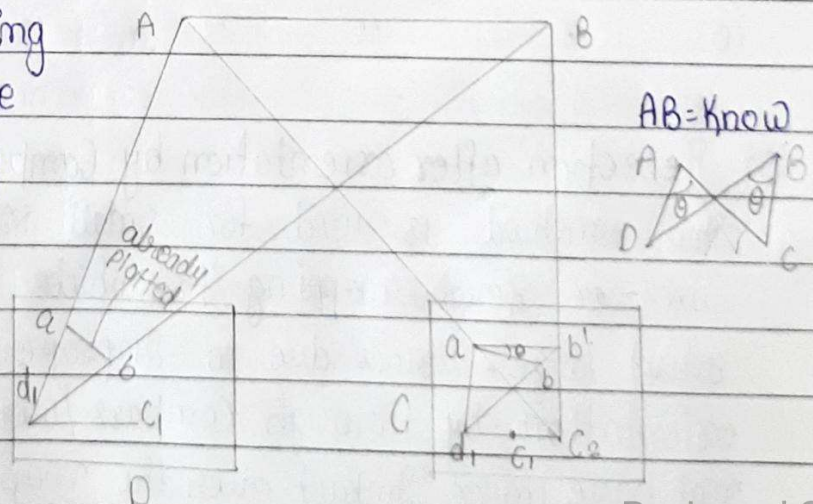
This method is used for small scale work or Rough mapping for which ~~total~~ large errors due to Improper orientation by through Compass/would not have major impact over the map.



- (B) Resection After Orientation By Backsight

-
- Diagram illustrating the projection of a line AB perpendicular to a horizontal plane. The top view shows a line segment $a-b$ and a point c_1 . The front view shows a line segment $a-b$ and a point c . Dashed lines connect a to a and b to b . A dashed line also connects c_1 to c .

Two Points Consists of locating the position of a plane table stations on plan / drawing sheet by observation of two well defined points, whose position is already being plotted on plan.



- Consider Another station near the station C in such a way that $\angle CAD$ and $\angle CBD$ will be neither acute nor obtuse
- set the plane table on D and shift it on table $[d_1]$ and do orientation as per eye judgement
 - Make ab parallel to AB and clamp the table
 - Pivot the alidade and sight A, and draw a Resector
 - Pivot the alidade and sight B and draw a Resector Point of intersection d_1 , this is approximate position of Point D on plane
 - Reasons being because orientation is not exact.
 - Pivot the alidade at d_1 and sight the station C and draw Ray d_1C_1 in which C_1 is Centering Corresponding C,
 - Put alidade along C_1d_1 and sight the Ranging Rod which is on D station
 - Pivot the alidade at a and sight the A and draw Resector the Resector comes out to be ac_2, c_2 indicate approximate position of C. Pivot the alidade at c_2 sight the B c_2b_1 , not Pass exactly through.

(D) RESECTION AFTER ORIENTATION BY 3 POINT PROBLEM.

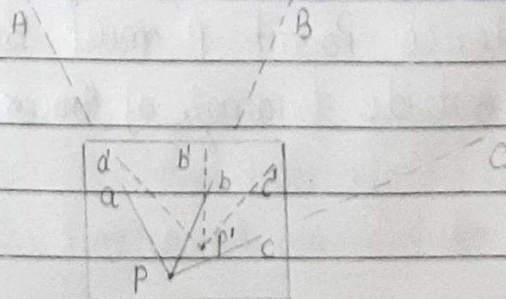
3 Point Problem consist of locating the position of instrument station on plane or drawing sheet with the help of observation of 3 well defined points whose position have been already plotted on the plan.

It can be done by following method

- (i) Mechanical method [Tracing paper method]
- (ii) Graphical method
- (iii) Lehmann's method [Trial and Error method]

(a) MECHANICAL METHOD.

- It was already drawn on drawing sheet, 3 Point.

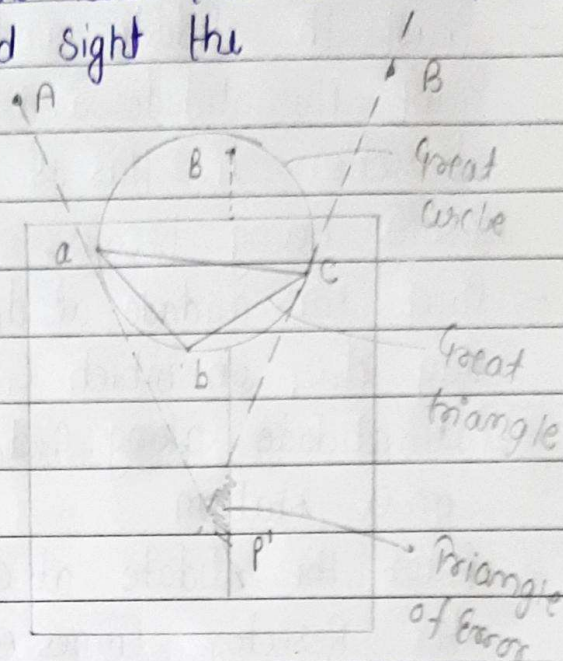


(II) GRAPHICAL METHOD: [BESSEL'S METHOD] : It is also termed as Inscribed Quadrilateral method.

(III) LEHMANN'S METHOD: [HIT AND TRIAL METHOD]:

- Set the table at P and Orient is approx such that ab is $\parallel$ to AB
- Keep the alidade pivoted about a, b, c and sight the station A, B, C respectively and draw the corresponding Resectors.

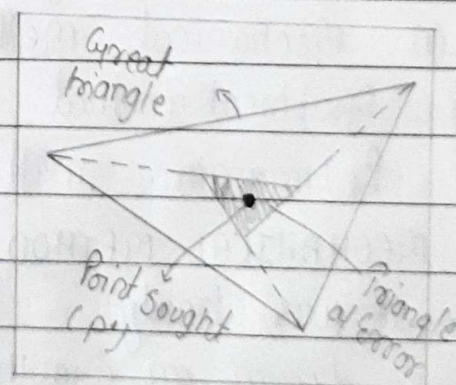
- If these Resectors meet at a point table is properly oriented but if they meet at 3 different points forming a triangle of error. It signifies ~~triangle~~ table is out of orientation.



- This triangle of error gives the idea of further orientation that is required and table would be properly oriented when the triangle of error is converted into a single point.
- To do so consider a point P' according to your choice as per LEHMANN Rule. Keep the alidade along P'a and turn the table to sight station A.
- Plot the alidade at b and c and sight B and C respectively and draw the Resectors.

Keep on repeating this process, upto an extent triangle of error is converted into single point.

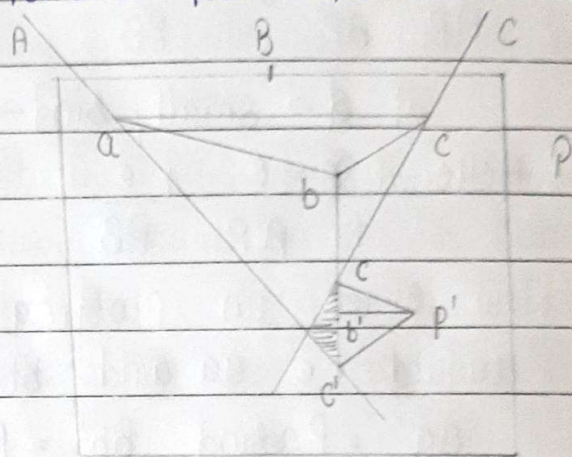
Lehmann's Rule: - If station P is outside the great triangle, the triangle of error also lies outside the great triangle, Hence Point P' must be considered outside triangle of error.



- on the same lines, if station P lies inside the great triangle, the triangle of error also lies inside the great triangle, Hence Point P' must also be constructed inside the triangle of error.

- The Point P should be so chosen that its distance from the Resectors (Aa, Bb, Cc) P'a', bP'b', PC' respectively must be in proportion to the distance of Point P from A, B and C

$$\frac{P'a'}{PA} = \frac{bP'b'}{PB} = \frac{PC'}{PC}$$



- The Point P' should be so chosen that it is on the same side [Left or Right] of all the Resectors Aa, Bb, Cc

ERROR IN PLANE TABLE

SURVEYING

- Error will occur if table board is not perfectly plane
- If fiducial edge is not straight, lines drawn would also not be straight
- If sight vanes are not, error will be observed in sighting.
- If magnetic compass is not proper, error will be found in orientation of table
- If level tube is not proper, the plane table would not be horizontal, when bubble is at the center.
- If magnetic compass is not proper, the plane table would not be horizontal
- If plumbing fork and bob is defective, centering of table is not proper, and error will occur whose magnitude is computed as follows

- Since the table is not centered properly the point P will not be over the station P. Hence linear error in centering P

- Angular Error in Centering

$$\gamma = \angle APB - \angle AP'B$$

$$\gamma = \alpha + \beta$$

In $\triangle APC$ and $B'DP$

$$\sin \alpha = \frac{PC}{AP}, \quad \sin \beta = \frac{PD}{PB}$$

$$\text{Si } \alpha = \frac{PC}{AP} = \frac{e}{AP}, \quad \beta = \frac{e}{PB}$$

if $e \rightarrow$ small, $\sin \alpha \rightarrow \alpha$

$$\text{Here } \gamma = \frac{e}{AP} + \frac{e}{PB}$$

Now error in plotting of Point A and B on plane table would be aa' and bb'

$$aa' = Pa \sin \alpha \quad bb' = Pb \sin \beta$$

$$Pa = S \cdot AP, \quad Pb = S \cdot PB \quad \text{Here } S \text{ is R.F.}$$

$$aa' = S \cdot AP \cdot \frac{e}{AP}, \quad bb' = S \cdot PB \cdot \frac{e}{PB}$$

$$aa' = S \cdot e$$

$$bb' = S \cdot e$$

Hence displacement of Point A and B on plane table from their correct position is Se .

Q1: In setting up of the plane table at a station "P" the corresponding plotted point P was not accurately centered over P. If the displacement of P was 8 cm in the dirⁿ at Right Angle to the ray, then determine the displacement of Pt. from its true position on the Plan if $S = 1/500$ (ii) $S = 1/50$

Solⁿ) $aa' \text{ or } bb' = Se$

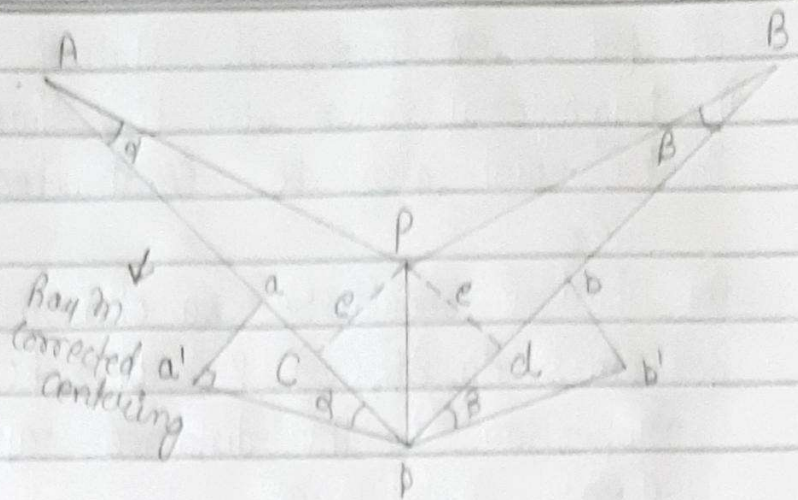
$$(ii) \frac{1}{50} \times 20 = 0.4 \text{ cm}$$

$$i) \frac{1}{500} \times 20 = 0.04 \text{ cm}$$

Note: It can be stated that centering error is significant when scale is large

- Accurate centering in plane table surveying necessary for large scale map

- Exact orientation is more important than accurate centering for small scale map.



THEORY OF ERRORS

In Surveying measurement of both linear distance and angular is to be carried out true value of which is not possible to measure as some or other error were there.

Following types of error are observed in measurement

(a) Mistake or Gross Error: These error occurs due to inexperience or carelessness of the surveyor

- This mistake are random in nature and can be eliminated by adopting standard operation procedure.

(b) Systematic or Cumulative Error: These error follow some pattern [i.e. magnitude and nature remain same for same condition]

- They can be computed very easily and suitable correction can be applied

- Surveying Instrument are designed such that these errors eliminated automatically

(c) Random / Accidental / Compensating Error: These error occur due to lack of perfection.

- These error tends to compensate each-other when large in No.

- These error follow law of chance or Probability Hence can be easily computed. As follow, The law of probability says, that error of small magnitude has higher probability of occurrence than the error of large or large magnitude.

- It follows normal probability distribution curve

$$\text{Error} = \text{Measured Value} - \text{True Value}$$

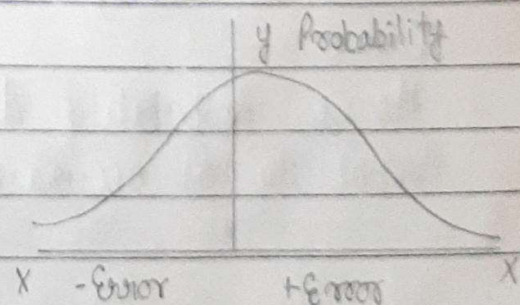
Since true value of the observation cannot be found instead most Probable value is used.

Most Probable Value: It is the value

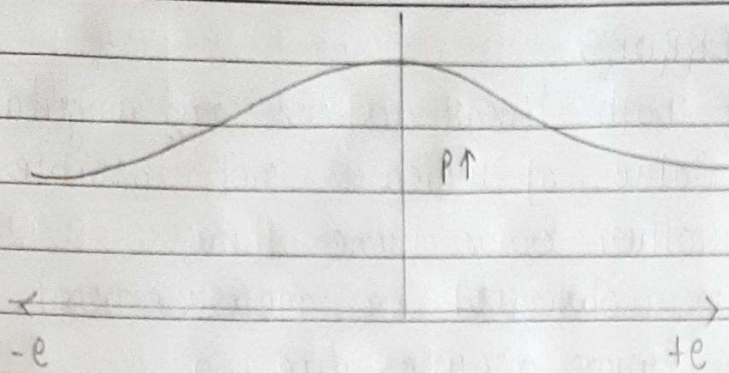
of the quantity which is closed to the true value than any other value

$$\text{Hence Error} = MV - MPV$$

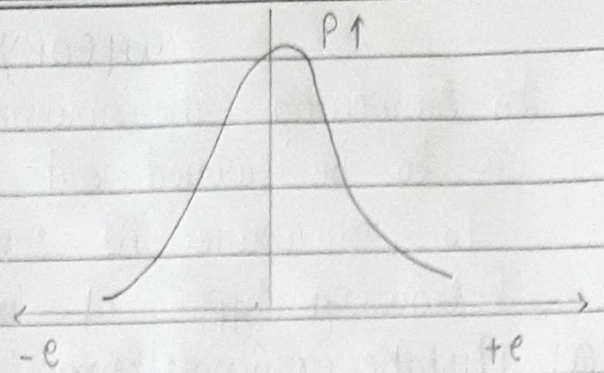
{ Residual }



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It represents large scatter of error
Hence signifies low precision



It represents small scatter of error
Hence signifies high precision.

NOTE: Accidental Error is since considered to follow Normal Probability distribution, it can be understood as

- The Normal frequency distribution curve is also termed as Gaussian Distribution Curve, which is given by

$$y = \frac{1}{\sqrt{2\pi}\sigma} e^{-1/2[v^2/\sigma^2]}$$

y = frequency of occurrence of Residual

σ = St. deviation

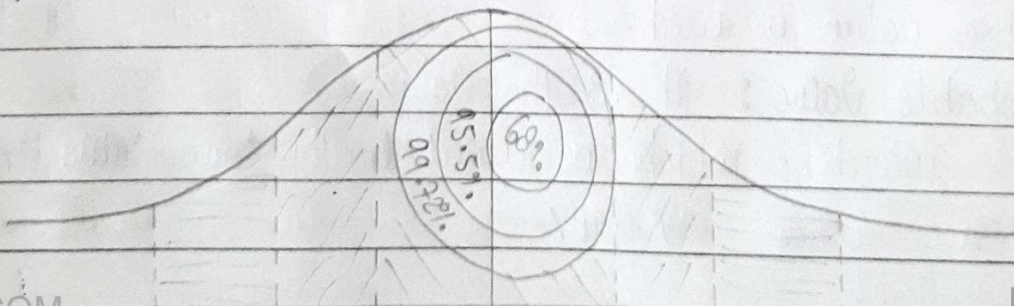
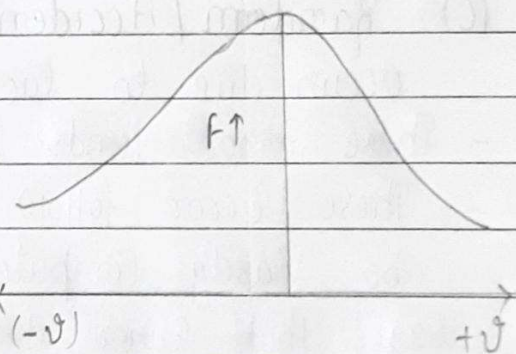
v = Residual = $MV - MPV$

$$\sigma = \sqrt{\frac{\sum v^2}{(n-1)}}$$

Here standard deviation may also be termed as root mean square error of measured value

If the Height of Normal Probability distribution curve is so adjusted that area under this curve is unity, it is termed as Normal Probability distribution Curve

- Hence Probability distribution Curve can be used to find the Probability of occurrence of an error in b/w two limits



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DEGREE OF PRECISION

- Here weightage of all the observation is considered equal.

(1) STANDARD DEVIATION: $\sigma = \sqrt{\frac{\sum v^2}{n-1}}$ $n = \text{no of observation mode.}$

(2) VARIANCE: It is square of standard deviation

$$V = \sigma^2 = [\sum v^2 / (n-1)]$$

(3) STANDARD ERROR OF SINGLE OBSERVATION: Standard Error of single observation is equal to standard deviation only

$$\sigma = \pm \sqrt{\frac{\sum v^2}{n(n-1)}} = \pm \frac{\sigma}{\sqrt{n}}$$

(4) STANDARD ERROR OF MEAN: Standard deviation of mean is termed as st. Error or mean

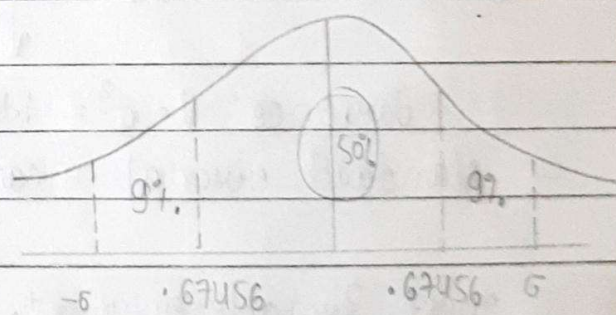
$$\sigma_m = \pm \sqrt{\frac{\sum v^2}{n(n-1)}} = \pm \frac{\sigma}{\sqrt{n}}$$

(5) MOST PROBABLE ERROR: It is defined as error for which there are 50% chances that the true error will be less than the Probable Error and 50% chances that the true value more than Probable Error
 most Probable Error = $\pm 0.6745 \sigma$

$$= \pm 0.6745 \sqrt{\frac{\sum v^2}{n-1}}$$

(6) MOST PROBABLE ERROR OF MEAN:

$$\sigma_m = \pm 0.6745 \sigma_m = \pm 0.6745 \sqrt{\frac{\sum v^2}{n(n-1)}}$$



(7) MAX ERROR: Maximum Error Corresponds to the Probability distribution Curve Extending to the Infinity, which Practically not Possible to Calculate Hence 99.9% is taken as max error.
 it corresponds to ~~3.95~~ $\pm 3.29 \sigma$

- Thus max error help to separate mistake from Random Error.

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- If any measurement deviates from the mean more than $\pm 3.29\sigma$ then measurement is considered as mistake and is rejected

Question) $\Rightarrow$ In a particular survey, the measurement of one of the distance are made by a group using same instrument find out
 (a) std deviation of each group (b) St error of mean (c) most Probable error of mean (w) most Probable error (v) Variance (vi) Max error

SNo.	1	2	3	4	5	6	7	8	9	10
Group(x)(cm)	75	76	78	74.5	76.5	75.5	77	77.5	74.6	75.4

Answer) $\text{mean} = \frac{760}{10} = 760$

$$\sum v^2 = (75-76)^2 + 0 + (78-76)^2 + (74.5-76)^2 + (76.5-76)^2 + (75.5-76)^2 + (77-76)^2 + (77.5-76)^2 + (74.6-76)^2 + (75.4-76)^2$$

$$\sum v^2 = 13.32$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\sum v^2}{n-1}} = \sqrt{\frac{13.32}{9}} = \sqrt{1.48} = 1.216$$

$$\text{Variance } V = \sigma^2 = 1.48$$

$$\text{Standard error of mean } \sigma_m = \sqrt{\frac{\sum v^2}{n(n-1)}} = \sqrt{\frac{13.32}{10(10-1)}} = .384$$

$$\text{most Probable error} = \pm .6745\sigma = \pm .6745 \times 1.216 = \pm .820$$

$$\text{most Probable error of mean} = .6745\sigma_m = .6745 \times .384 = .259$$

$$\text{Max Error} = \pm 3.29\sigma = \pm 3.29 \times 1.216 = \pm 4$$

Uptil Now, the weightage of every observation was same But it is changed analysis is done as follow

- Here weightage of a quantity Represent / signifies its Relative trust worthness or Indicates its Relative Precision

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- Weightages always Express As no, Higher is the weightage higher is the Precision and lower is weightage, lower is Precision
- weightage are allocated to the observation A/c to following Rules.

(i) Weightage are assigned as Per degree of Precision, Hence weightage are taken Inversely Proportional to Variance

$$\frac{W_1}{W_2} = \frac{V_2}{V_1}$$

(ii) Weightage of the quantities measured in similar Condition is directly proportional to no of times the quantity is being measured (n)

(iii) Weightages are also being Provided to the observation By the Survey on the Basis of Experience as per Prevailing field Condition / Environmental Condition.

(iv) Weightage of the level line is Inversely Proportional to length of line

$$\frac{W_1}{W_2} = \frac{L_2}{L_1}$$

While Assigning the weightages to the observer following laws of weightage to be applied

(1) The weightage of AM of no of observation of some unit weightage is Equal to either summation of their unit weightage or no of observation.

(2) The weight of weighted AM of no of observation of diff weightage is Equal to same of their individual weightage

(3) The weightage of algebraic summation or algebraic subtraction of two or more observation is Equal to Reciprocal of summation of reciprocal of their individual weight

for Example: $A = 10.5m$ $W_A = 2$ $B = 12m$ $W_B = 3$

$$W(A+B) = \frac{1}{\frac{1}{2} + \frac{1}{3}} = \frac{6}{5}$$

$$W(A-B) = \frac{1}{\frac{1}{3} + \frac{1}{2}} = \frac{6}{5}$$

- (4) If the quantity of given weightage is multiplied by a factor than the weightage of the Result is obtained by dividing the given weightage By square of the factor

Ex - $A = 10m$ $W_A = 2$

$$3A = 30m \quad W_{3A} = \frac{W_A}{3^2} = \frac{2}{9}$$

- (5) If the quantity of the given weightage is divided by a factor, than the weightage of Result is obtained by multiplying the given weight by square of that factor.

- (6) If an Equation is multiplied by its own wt. than the weightage of Resultant Equation is equal to Reciprocal of weightage of the given Equation

- (7) The weightage of an Eqⁿ remain same if all signs of Equation are changed

$$A+B = 30$$

$$W_{(A+B)} = 2$$

$$-A-B = -30m$$

$$W_{(-A-B)} = 2$$

- (8) The weightage of an Equation remain unchanged if a Constant is added or subtracted from it

$$A+B = 80^\circ, \quad W_{(A+B)} = 2$$

$$180 - (A+B) = 100, \quad W_{180-(A+B)} = 2$$

OR $180 + (A+B) = 260^\circ, \quad W_{180+(A+B)} = 2$

INDICES OF PRECISE OF OBSERVATION HAVING DIFFERENT WEIGHTAGE

- (i) standard deviation of weighted observation

$$\sigma_w = \sqrt{\frac{\sum (w_i v_i^2)}{n-1}}$$

- (ii) standard deviation of mean of weight observation or Error

$$(\sigma_m)_w = \pm \sqrt{\frac{\sum w_i v_i^2}{(\sum w_i)^2}}$$

(iii) Standard Error of Single observation of Weightage

$$(\sigma_1)_w = \pm \sqrt{\frac{\sum (wv)^2}{(n-1)w}}$$

(iv) Most Probable Error of Single observation

$$\rho_w = \pm 0.6745 (\sigma_1)_w$$

Question: The following are the measured value of angle P, with the corresponding weightage.

$\angle P$	w	(a) Determine standard deviation
1) $81^\circ 30' 20''$	2	(b) Standard Error of Weightage mean
2) $81^\circ 30' 25''$	3	(c) Standard Error of single observation
3) $81^\circ 30' 28''$	2	(d) Probable error of weighted mean.

Solution)	$\angle P$	w_i	v_i	$(v_i)^2$	wv_i^2
	$81^\circ 30' 20''$	2	-4.4"	19.36"	38.72"
	$81^\circ 30' 25''$	3	+0.6"	0.36"	1.08"
	$81^\circ 30' 28''$	2	+3.6"	12.96"	25.92"

$$MPV = \frac{2 \times 81^\circ 30' 20'' + 3 \times 81^\circ 30' 25'' + 2 \times 81^\circ 30' 28''}{2+3+2}$$

$$= 81^\circ 30' 24.4''$$

or

$$MPV = 81^\circ 30' + \frac{20'' \times 2 + 25'' \times 3 + 28'' \times 2}{2+3+2} = 81^\circ 30' 24.4''$$

(i) standard deviation $\sigma_w = \sqrt{\frac{\sum wv^2}{n-1}} = \sqrt{\frac{65.72}{2}} = 5.73 \text{ sec}$

(ii) standard Error of wt. mean $\pm \sqrt{\frac{\sum wv^2}{(n-1)\sum w}} = \sqrt{\frac{65.72}{2 \times 7}} = 2.166''$

(iii) $(\sigma_1)_w = \sqrt{\frac{\sum wv^2}{(n-1)w_1}} = \frac{5.73}{\sqrt{3}} = 3.308''$

(iv) $\rho_{mw} = \pm 0.6745 \sigma_{mw} = \pm 0.6745 \times 2.16$
 $= \pm 1.45692''$

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PROPAGATION OF STANDARD ERROR

Let a quantity is dependent upon several other quantities

$$A = f(x, y, z)$$

Here total error in A is given by

$$\delta A = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y + \frac{\partial f}{\partial z} \delta z$$

 $\delta x, \delta y, \delta z$ are individual error in x, y, z and standard error in A is given by

$$e_A^2 = \left(\frac{\partial f}{\partial x} e_x \right)^2 + \left(\frac{\partial f}{\partial y} e_y \right)^2 + \left(\frac{\partial f}{\partial z} e_z \right)^2$$

 e_x, e_y, e_z are standard error in x, y, z
for e_x :

$$A = x \pm y$$

$$e_A^2 = \left(\frac{\partial f}{\partial x} e_x \right)^2 + \left(\frac{\partial f}{\partial y} e_y \right)^2$$

$$= \left[\frac{\partial (x \pm y)}{\partial x} e_x \right]^2 + \left[\frac{\partial (x \pm y)}{\partial y} e_y \right]^2$$

$$e_A = [e_x^2 + e_y^2]^{1/2}$$

for e_x :

$$A(f) = xyz$$

$$e_A^2 = \left(\frac{\partial f}{\partial x} e_x \right)^2 + \left(\frac{\partial f}{\partial y} e_y \right)^2 + \left(\frac{\partial f}{\partial z} e_z \right)^2$$

$$= (yz e_x)^2 + (xz e_y)^2 + (xy e_z)^2$$

$$= (xyz)^2 \left\{ \left(\frac{e_x}{x} \right)^2 + \left(\frac{e_y}{y} \right)^2 + \left(\frac{e_z}{z} \right)^2 \right\}$$

$$e_A = xyz \left[\left(\frac{e_x}{x} \right)^2 + \left(\frac{e_y}{y} \right)^2 + \left(\frac{e_z}{z} \right)^2 \right]^{1/2}$$

for e_x :

$$e_A^2 = \left(\frac{1}{y} e_x \right)^2 - \left[\frac{x}{y^2} e_y \right]^2 \Rightarrow \left[\frac{e_x}{y} \right]^2 + \left[\frac{-x}{y^2} e_y \right]^2$$

$$= x^2 \left[\left(\frac{e_x}{x} \right)^2 + \left(\frac{e_y}{y} \right)^2 \right]$$

$$\frac{x}{y} \left[\left(\frac{e_x}{x} \right)^2 + \left(\frac{e_y}{y} \right)^2 \right]^{1/2}$$

For e_x :

$$A(f) = x^n$$

$$e_A^2 = (n x^{n-1} e_x)^2$$

$$e_A = n x^{n-1} e_x$$

Question: If Angle A is related to Angle B and C. As follows
 $A = 4B \times C$. The observed values of B and C and their
 corresponding standard error as follow

$$B = 20'' \pm 0.03''$$

$$C = 30'' \pm 0.04''$$

Determine the standard error of A

Answer)

$$A = 4B \times C$$

$$e_A^2 = \left[\frac{\partial (4B \times C)}{\partial B} e_B \right]^2 + \left[\frac{\partial (4B \times C)}{\partial C} e_C \right]^2$$

$$e_A^2 = \left[(4C e_B)^2 + (4B e_C)^2 \right]$$

$$= \left[4 \times (30 \times 0.03) \right]^2 + \left[4 \times (20 \times 0.04) \right]^2$$

$$e_A = \pm 4.81$$

Question: In measuring Round Angle at station "O" the 3rd angle C, closing the horizon is found from the Relation, $C = 360^\circ - (A + B)$ if Angles A and B have standard error of ± 3 sec. and ± 4 sec. respectively determine the st. error of C

$$\text{Answer) } e_C^2 = \left[\frac{\partial [360^\circ - (A+B)]}{\partial A} e_A \right]^2 + \left[\frac{\partial [360^\circ - (A+B)]}{\partial B} e_B \right]^2$$

$$e_C^2 = (-e_A)^2 + (e_B)^2$$

$$e_C^2 = (3'')^2 + (4'')^2$$

$$e_C = \sqrt{25} = 5''$$

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Rules for applying Correction

- (1) If all observation are of same weightage the error is distributed equally in all the observations, Hence Correction is also equal
- (2) Correction are applied to the observation Inversely Proportional to weightage
- (3) Correction applied to the observation is directly Proportional to square of st. deviation or variance
- (4) In Case of level line the Correction is proportional to length of line

Question: Adjust the Angle A, B, C and d which close the horizon

Angle	Weightage
$\angle A = 100^\circ 30' 22''$	1
$\angle B = 80^\circ 40' 10''$	2
$\angle C = 90^\circ 20' 08''$	3
$\angle D = 88^\circ 29' 25''$	4

Answer) $\angle A + \angle B + \angle C + \angle D = 360^\circ 0' 5''$

$$\text{Sum of angle} = (2n-4) 90 = 360^\circ$$

$$\text{Error of closure} = 5''$$

$$\text{Correction} = \pi - m_v = -5''$$

$$C_1 : C_2 : C_3 : C_4 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 12' : 6' : 4' : 3'$$

$$C_1 = \frac{12 \times (-5'')}{12+6+4+3} = -2.4 \text{ sec}$$

$$C_2 = 1.2 \text{ sec}, \quad C_3 = \frac{4(-5'')}{25} = -.8'', \quad C_4 = \frac{3(-5'')}{25} = -.6''$$

$$\angle A = 100^\circ 30' 22'' - 2.4 = 100^\circ 30' 19.6''$$

$$\angle B = 80^\circ 40' 10'' - 1.2'' = 80^\circ 40' 8.8''$$

$$\angle C = 90^\circ 20' 08'' - .8'' = 90^\circ 20' 7.2''$$

$$\angle D = 88^\circ 29' 25'' - .6'' = 88^\circ 29' 24.4''$$

THEORY OF LEAST SQUARE

According to this theory "MPV" of the observation of equal weightage is that variation (v) to minimum value

$$\sum v^2 = \text{minimum}$$

$$(x_1 - \text{MPV})^2 + (x_2 - \text{MPV})^2 + \dots + (x_n - \text{MPV})^2 = \sum v^2$$

For summation to be minimum of $\sum v^2$

$$= \frac{\partial \sum v^2}{\partial \text{MPV}} = 0$$

$$= -2(x_1 - \text{MPV}) - 2(x_2 - \text{MPV}) - \dots - 2(x_n - \text{MPV}) = 0$$

$$-2[x_1 + x_2 + \dots + x_n - n \text{MPV}] = 0$$

$$\text{MPV} = \frac{x_1 + x_2 + \dots + x_n}{2n}$$

If the observation are of unequal wt. the MPV is the value which makes the same of the product of wt and square of Residual to be min.

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$$\sum wv^2 = \text{minimum}$$

$$\text{Hence, MPV} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{(w_1 + w_2 + \dots + w_n)}$$

The least square Adjustment is done by Normal Equation method. It may also be termed as observation Eqⁿ method which is formed as follow

CASE 1: FOR OBSERVATION OF EQUAL WEIGHT

To form the normal Equation in this case for each of the unknown quantity, multiply each observation Eqⁿ with the algebraic Coeff of that unknown quantity and add.

For Ex: Find the Normal Eqⁿ of x, y, z from the following Equation of Equal wt and determine MPV of x, y, z

$$4x + 2y + z - 11 = 0$$

$$3x + 3y + 2z - 9 = 0$$

$$5x + y + 3z - 16 = 0$$

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Answer) For Normal Eqⁿ of X

$$4[4x+2y+z-11] + 3[3x+3y+2z-9] + 5[5x+y+3z-16] = 0$$

$$50x + 22y + 25z - 151 = 0 \quad \text{--- (A)}$$

For Normal Eqⁿ of Y

$$2[4x+2y+z-11] + 3[3x+3y+2z-9] + 1[5x+y+3z-16] = 0$$

$$22x + 14y + 11z - 65 = 0 \quad \text{--- (B)}$$

For Normal Eqⁿ of Z

$$1[4x+2y+z-11] + 2[3x+3y+2z-9] + 3[5x+y+3z-16] = 0$$

$$25x + 11y + 11z - 77 = 0 \quad \text{--- (C)}$$

Normal Eqⁿ of all these unknown quantities is solved to find the MPV of these quantities

So from Normal Equation A, B, C of X, Y, Z respectively

$$\text{MPV} \Rightarrow \begin{cases} X = 2.67 \\ Y = -0.33 \\ Z = 1 \end{cases}$$

CASE 2: FOR OBSERVATION OF UNEQUAL WEIGHT

To form the Normal Equation for each of the unknown quantity, multiplying each observation equation with the algebraic coeffs and weightage of the observation and add

For Ex: Data same as previous problem with weight 2, 3, 4 respectively for three observation equation

Ans) For X

$$4 \times 2[4x+2y+z-11] + 3 \times 3[3x+3y+2z-9] + 5 \times 4[5x+y+3z-16] = 0$$

$$159x + 63y + 86z - 489 = 0$$

For Normal Eqⁿ of Y

$$2 \times 2[4x+2y+z-11] + 3 \times 3[3x+3y+2z-9] + 1 \times 4[5x+y+3z-16] = 0$$

$$63x + 39y + 34z - 189 = 0$$

For Normal Eqⁿ of Z

$$1 \times 2 [4x + 2y + z - 11] + 2 \times 3 [3x + 3y + 2z - 9] + 3 \times 4 [5x + y + 3z - 16] = 0$$

$$8x + 4y + 2z - 22 + 18x + 18y + 12z - 54 + 60x + 12y + 36z - 192 = 0$$

$$86x + 32y + 50z - 268 = 0$$

$$\text{MPV} = \begin{cases} x = 2.667 \\ y = -0.333 \\ z = -1 \end{cases}$$

Note: In the previous cases unknown quantities are dependent on each other (X, Y, Z), if they are independent then Normal Equation are found formed by considering given equation [Independent] as observation equation

For Example: Find MPV of angles θ and ϕ from the following observation Eqⁿ

$$\theta = 40^\circ 15' 2'' \quad \phi = 45^\circ 12' 18'' \quad \theta + \phi = 85^\circ 27' 45''$$

Answer) For Normal Eqⁿ of θ

$$1 \times 1 \times \theta = 40^\circ 15' 2'' \quad \text{--- (1)}$$

$$1 \times 1 \times (\theta + \phi) = 85^\circ 27' 45'' \quad \text{--- (2)}$$

$$(1) + (2)$$

$$2\theta + \phi = 125^\circ 43' 6''$$

For Normal Equation of ϕ

$$1 \times 1 \times \theta = 45^\circ 12' 18'' \quad \text{--- (3)}$$

$$1 \times 1 \times (\theta + \phi) = 85^\circ 27' 45'' \quad \text{--- (4)}$$

$$(3) \times (4)$$

$$\theta + 2\phi = 130^\circ 40' 30''$$

$$\theta = 40^\circ 15' 23''$$

$$\phi = 45^\circ 12' 20''$$

(Question) Find the MPV of angle θ from the given observation Eqⁿ

$$\theta = 40^\circ 20' 15''$$

$$2\theta = 80^\circ 40' 50''$$

$$4\theta = 161^\circ 21' 40''$$

wt
2

3

1

Answer) Normal Eqⁿ of θ

$$1 \times 2\theta = 80^\circ 40' 30''$$

$$2 \times 3 \times 2\theta = 484^\circ 5' 0''$$

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$$4 \times 40 = 645^{\circ} 26' 40''$$

$$\text{Add all } 300 = 1210^{\circ} 12' 10''$$

$$\text{MPV of } \theta = 40^{\circ} 20' 24.33''$$

- # (1) The Probability Error or st. Error of levels is directly proportional to $\sqrt{L}$ length of level line is $e \sqrt{L}$ — (i)
- (2) The weightage of levels in any level line is inversely proportional to length of the line

$$w \propto \frac{1}{L} \text{ — (ii)}$$

from (1) and (2) $w \propto 1/e^2$

Degree of accuracy in angular measurement = $\frac{\text{Angular Error of closure}}{\sqrt{\text{No of Angle Measured}}}$

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Ex: If Angular Error of closure is $180^{\circ} 23' 3''$ in the measured measurement of 9 angles, find the degree of Accuracy in angular measurement.

Answer) $\text{DoA} = \frac{180'}{\sqrt{9}} = 60''$

PHOTOGRAMMETRY

It is the science of obtaining, physical object through the process of Recording, measuring and Interpreting Photographs

If the Photograph is taken from a Camera mounted on an aircraft it is termed as "AERIAL PHOTOGRAMMETRY"

If the same Camera is mounted on or above the ground it is termed as Ground Photogrammetry.

Note: If vertical features of Mountain, cliffs, Valley is required Ground Photogrammetry is preferred, But if alignment of Railways, Roadways, Canal, pipeline is ~~studied~~ studied Aerial photogrammetry is preferred.

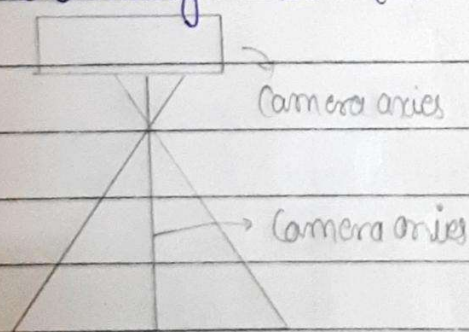
- Aerial photograph are further classified into two

(A) Vertical photographs

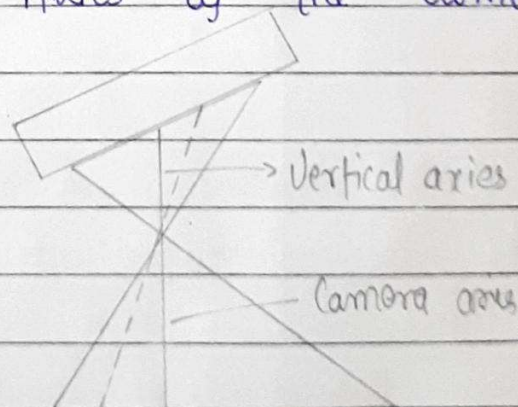
(B) oblique photographs.

- In Vertical photographs, Camera Axis is Vertical i.e line of gravity of Camera and axis of Camera Coincides. However there can be a difference of $1-3^\circ$

- In oblique photograph, Camera Axis is Inclined Considerably to the Vertical Axis of the Camera



Vertical photograph

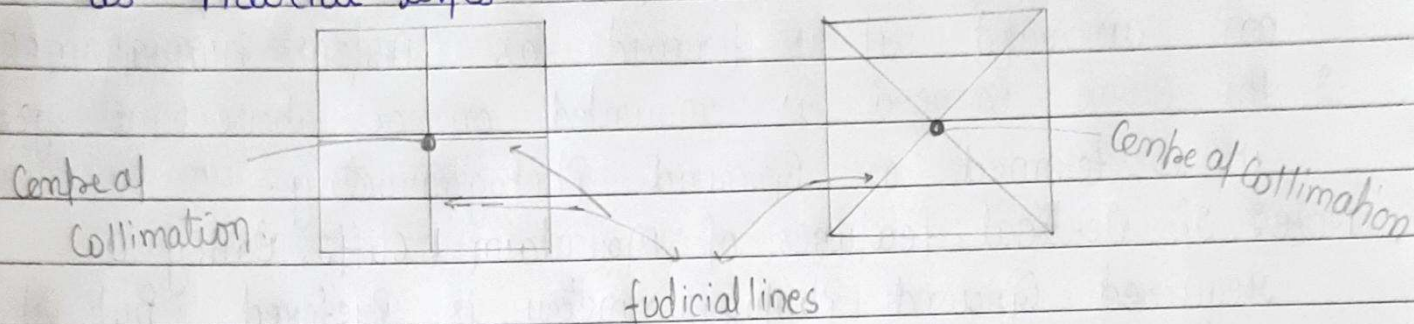


oblique photograph

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Note: Reference Point located in the ~~frame~~ image plane of the camera to provide the reference line for measurement of image distance is termed as fiducial mark
[Generally they 4 or 8 in no]

- Lines joining the opposite fiducial mark are termed as fiducial lines

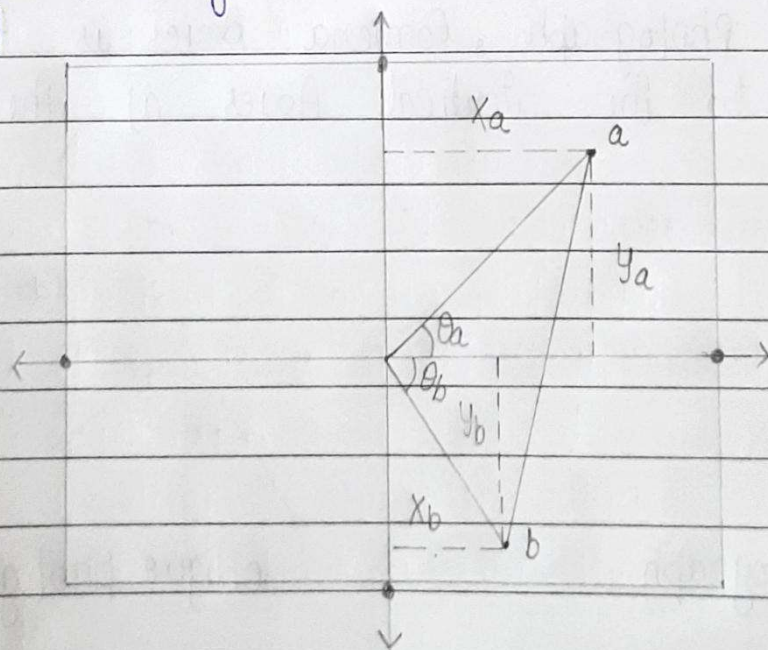


PHOTOCOORDINATE MEASUREMENT

Photo Coordinate is generally a Rectangular Coordinate System, which is use for measuring distance on the photograph.

- Positive X-axis is usually taken along the fiducial line, which is nearly parallel to direction of the light and (+)ve y-axis is taken at 90° to it in Anticlock-wise direction

- The Coordinate of Image Point a and b are then Marked as $[X_a, Y_a]$



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Here the distance b/w the two points a and b is given by

$$ab = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

$$\text{angle } ab = \theta_a + \theta_b = \tan^{-1}\left[\frac{y_a}{x_a}\right] + \tan^{-1}\left[\frac{y_b}{x_b}\right]$$

Note: In the Above Analysis the photoordinates are measured w.r.t fiducial line, But it can also be measured w.r.t fiducial mark and this method is termed as "ALTERNATIVE METHOD".

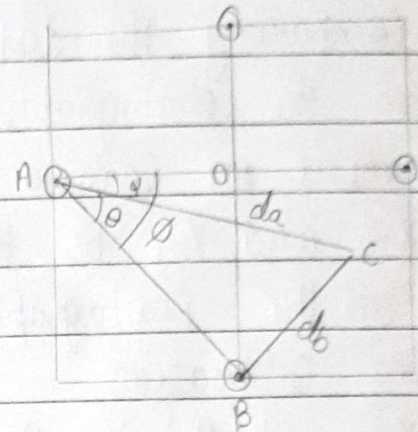
$$\cos \theta = \frac{d_a^2 + AB^2 - d_b^2}{2 d_a \cdot AB}$$

$$\phi = \tan^{-1}\left[\frac{y_b}{x_a}\right]$$

$$\alpha = \phi - \theta$$

$$y_c = d_a \sin \alpha$$

$$x_c = x_a + d_a \cos \alpha$$



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Example: In the Previous fig Calibrated Coordinate of fiducial mark A and B are as follow y
 Determine the Coordinate of Point C, if
 The measured distance d_a and d_b are
 Point A -115.0mm 0
 Point B 0 -115.0mm 184.32mm and 95.84mm Respectively.

Answer) $AB = \sqrt{(-115-0)^2 + (-115-0)^2} = 162.63 \text{ mm}$

$$\cos \theta = \frac{d_a^2 + AB^2 - d_b^2}{2 d_a AB} = \frac{184.32^2 + 162.63^2 - 95.84^2}{2 \times 184.32 \times 162.63}$$

$$\cos \theta = 0.85$$

$$\theta = 31.28^\circ$$

$$\phi = \tan^{-1}\left[\frac{-115}{-115}\right] = 45^\circ$$

$$Y_c = d \sin \alpha = -184.32 \sin 13.72^\circ = -43.72 \text{ mm}$$

$$X_c = X_A + d \cos \alpha = -115 + 184.32 \cos 13.72^\circ$$

$$X_c = 64.06 \text{ mm}$$

Note 2: Several Corrections are also being applied over the image distance in Photo Coordinate system.

(a) Shrinkage Corrections: The photograph may shrink or expand due to temperature

- The error due to shrinkage or expansion is found with the help by comparing the distance measured between ^{opp} fiducial mark from the photograph with the corresponding values determine in Camera Calibration

- Let X_m and Y_m be the distance measured of the fiducial mark on the photograph and X_c and Y_c be distance obtained during Calibration

- Let X_a' and Y_a' be the measured Co-ordinate of concerned point on image hence its corrected Co-ordinate X_a, Y_a

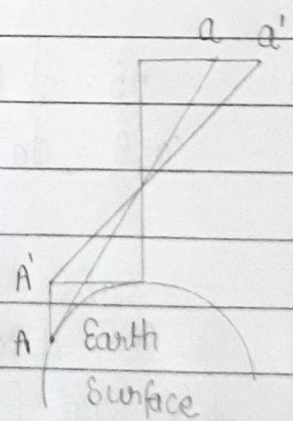
$$\frac{X_a}{X_a'} = \frac{X_c}{X_m} \Rightarrow X_a = \left[\frac{X_c}{X_m} \right] X_a'$$

$$\frac{Y_a}{Y_a'} = \frac{Y_c}{Y_m} \Rightarrow Y_a = \frac{Y_c}{Y_m} \times Y_a'$$

(B) Correction due to Earth Curvature

Position of Point A on the ground surface is represented by point "a" on the photograph

- If the effect of curvature of the Earth is not considered, Pt A on the ground surface corresponding to point A', which is represented by point a on the photograph



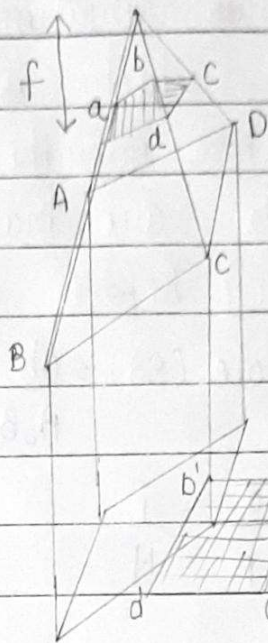
- Hence a correction of aa' is required due to the curvature of Earth [Here Correction of Curvature of Earth in levelling is AA']

SCALE OF VERTICAL PHOTOGRAPH

Scale of Photograph is defined as Ratio of Photodistance and ground distance.

$$\text{Scale of Photograph (S)} = \frac{\text{Photo distance}}{\text{Ground distance}}$$

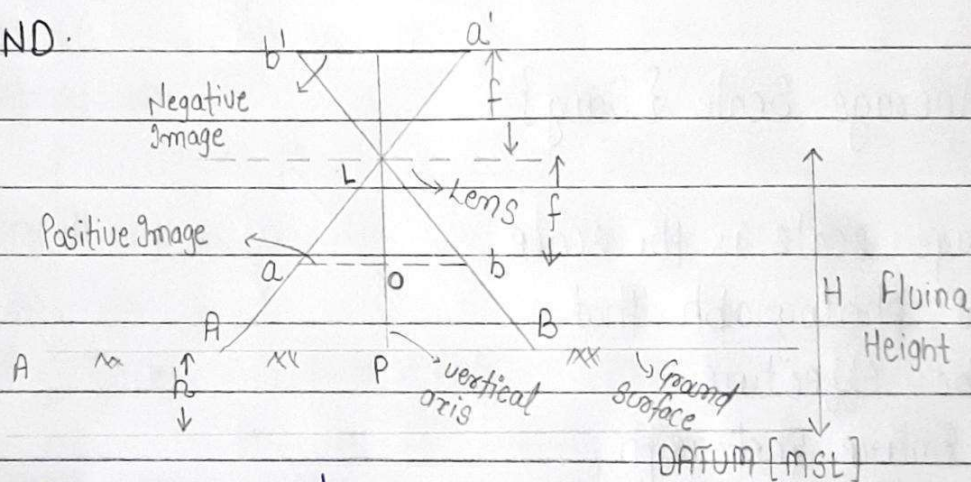
Projection on
Photograph [Perspective
Projection]



Projection on
map orthogonal
Projection

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FLAT GROUND.



$$\text{Scale of Photograph} = \frac{ab}{AB}$$

$$\Delta LO \sim \Delta LAP$$

$$\frac{LO}{AP} = \frac{LO}{LP} \Rightarrow \frac{ab/2}{AB/2} = \frac{f}{H'}$$

$$S = \frac{ab}{AB} = \frac{f}{H-h}$$

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Note: If the flying height H and focal length " f " of the camera is constant the scale of the photograph varies with the elevation of terrain " h " above the "MSL"

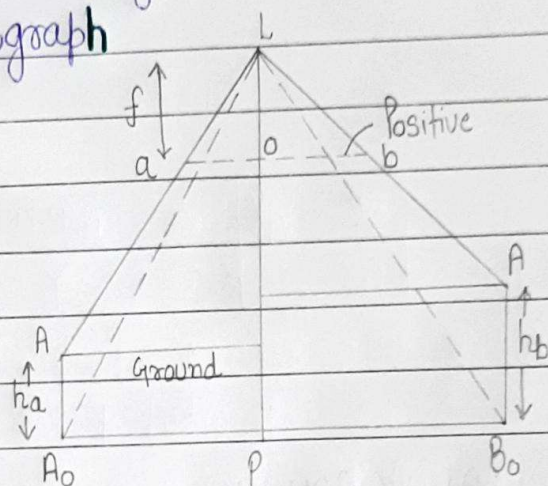
- Hence when Elevation is Higher, Scale is larger and when Elevation is lower, scale is smaller.
- To Eliminate this variation of scale of vertical photograph, Datum scale is used, which is the scale of vertical photograph that would be effective over the entire ground photograph if all the ground points are transferred vertically downward on the Reference [MSL] before being photographed.

$$\text{Datum scale (Sd)} = \frac{ab}{A_0B_0}$$

$$\Rightarrow \frac{L_0}{L_p} = \frac{f}{H}$$

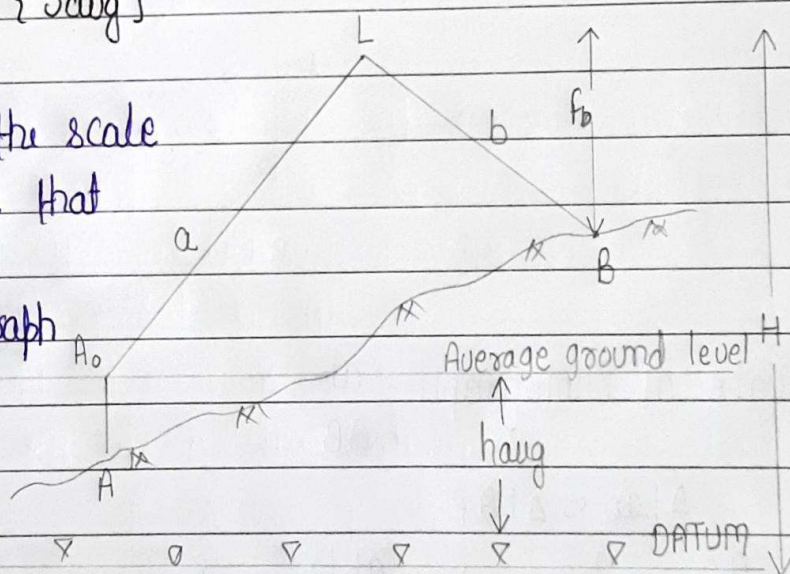
$$S_d = \frac{f}{H}$$

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Note: Average Scale { S_{avg} }

The average scale is the scale of vertical photograph that would be effective over the entire photograph if all the ground points were projected vertically either downward



or upward on the plane of Avg Elevation of the ground before being photographed

$$S_{avg} = \frac{f}{H - h_{avg}}$$

Question: The Elevation of three points X, Y, Z above the datum respectively are 2000m, 1500m and 3000m. If the flying height above the datum is 5000m determine the max and min scale also compute the avg scale if focal length of camera is 150mm.

Ans: Scale of photograph at Pt X, $S_x = \frac{f}{H-h_x}$

$$\Rightarrow \frac{150 \times 10^{-3}}{5000 - 2000} \Rightarrow S_x = \frac{1}{20,000}$$

$$\text{on Y, } S_y = \frac{f}{H-h_y} = \frac{150 \times 10^{-3}}{5000 - 1500} \Rightarrow \frac{1}{9333.33}$$

$$\text{on Z, } S_z = \frac{f}{H-h_z} = \frac{150 \times 10^{-3}}{5000 - 3000} = \frac{1}{13333.33}$$

$$S_{\text{avg}} = \frac{150 \times 10^{-3}}{5000 - 2166.67} = \frac{1}{18888.886}$$

Question: A map of the area plotted at the scale of 1:1000 is available. If the length of a runway on the map is 1500mm, determine the scale of the photograph. The photodistance of the runway is 200mm.

Ans: $S = \frac{1}{10000}$ scale of photograph = $\frac{\text{Photo dis}}{\text{Ground dis}} \quad \text{--- (1)}$

scale of map = $\frac{\text{map distance}}{\text{Ground distance}} \quad \text{--- (2)}$

from (1) and (2)

$$\frac{\text{scale of photograph}}{\text{scale of map}} = \frac{\text{photodistance}}{\text{map distance}}$$

$$\frac{200}{150} \times \frac{1}{1000} \Rightarrow \text{scale of photograph} = \frac{1}{7500}$$

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GROUND COORDINATE

$$\Delta Laa' \approx \Delta LAA'$$

$$\frac{x_a}{X_A} = \frac{H-h_a}{f}$$

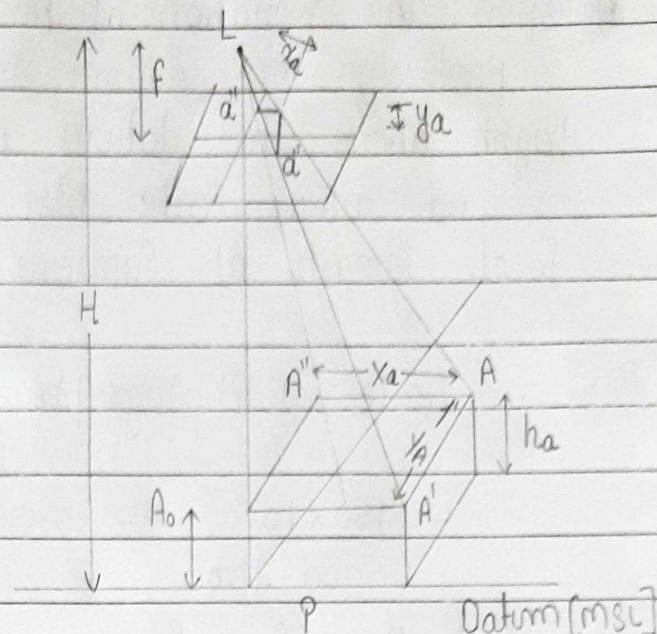
$$X_A = x_a \left[\frac{H-h_a}{f} \right]$$

Here $\frac{f}{H-h_a}$

= Scale of photograph at A SA

$$\therefore X_A = \frac{x_a}{SA}$$

$$\Delta Laa' \approx \Delta LAA' \Rightarrow \frac{y_a}{Y_A} = \frac{H-h_a}{f} \Rightarrow \boxed{Y_A = \frac{y_a}{SA}}$$



- The ground Co-ordinates are measured w.r.t X-axis and Y-axis
- The PhotoCoordinates are measured w.r.t fiducial line
- Let Ground Coordinate of Pt "A" is X_A and Y_A and Corresponding PhotoCoordinate are x_a and y_a

Question 3 → The measured PhotoCoordinate of Image Point a and b of two ground Point A and B are $x_a = 45\text{mm}$, $y_a = 38\text{mm}$,

$x_b = -40\text{mm}$, $y_b = -45\text{mm}$. Determine the ground Co-ordinate of Pt A and B and horizontal length of line AB

The Elevation of Point A and B are 200m and 150m Respectively flying height is 1500m above the datum Assume focal length $f = 150\text{mm}$.

Answer $X_A = x_a \left[\frac{H-h_a}{f} \right] = 45 \left[\frac{1500-200}{150 \times 10^{-3}} \right] = 390\text{m}$

$$Y_A = y_a \left[\frac{H-h_a}{f} \right] = 38 \left[\frac{1500-200}{150 \times 10^{-3}} \right] = 329.33\text{m}$$

$$X_b = x_b \left[\frac{H-h_b}{F} \right] = -40 \left[\frac{1500-150}{150 \times 10^3} \right] = -360$$

$$Y_b = -405$$

$$AB = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2} = \sqrt{(390 + 360)^2 + (329.33 + 405)^2}$$

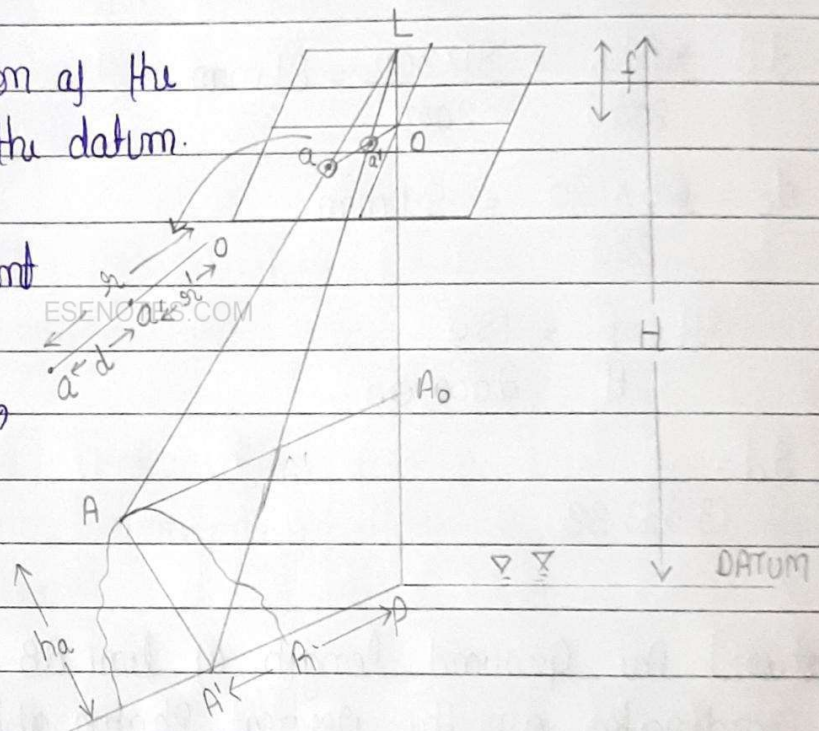
$$(AB = 1049.65)$$

RELIEF DISPLACEMENT

Relief displacement is the lateral displacement or drift in the photographic position of the image.

Note: Relief is the Elevation of the object above or below the datum.

- If object is above the datum Relief displacement is outward and if object is below the datum, Relief displacement is inward.



$$\Delta LAO \cong \Delta LAA_0$$

$$\frac{r}{R} = \frac{f}{H-ha}$$

$$FR = r(H-ha) \quad (1)$$

$$\text{In } \Delta LA'O \cong \Delta LA'P$$

$$\frac{r'}{R} = \frac{f}{H}$$

$$FR = r'H \quad (2)$$

from (1) and (2)

$$r(H-ha) = r'H \Rightarrow rha = rH - r'H$$

$$H = \frac{rha}{(r-r')} \Rightarrow \frac{rha}{H} = r - r'$$

Here $r - r' = d = \text{Relief displacement}$

$$\boxed{\frac{d = rha}{H}}$$

Question] If the given situation on vertical photograph taken at a flying height of 200m above the datum shows a triangular plot ABC of land. The radial distance of 3-points from the Principle Point are respectively 90mm, 81mm, 68mm. The elevation of points above the datum 260m, 200m, 150m respectively determine the relief displacement of all the 3 points and compute the datum scale. Assume focal length of camera = 150.

Ans] $dA = \frac{r_A h_A}{2000} = 11.7 \text{ mm}$

$dB = \frac{r_B h_B}{2000} = \frac{81 \times 200}{2000} = 8.1 \text{ mm}$

$dc = \frac{68 \times 150}{200} = 5.1 \text{ mm}$

$S_d = \frac{f}{H} = \frac{150}{2000.00}$

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$S_d = \frac{1}{13333.33}$

Question] The ground length of line AB is 100m, the measured coordinates on the vertical photograph $X_A = 18.35 \text{ mm}$, $Y_A = -62.4 \text{ mm}$, $X_B = 106.41 \text{ mm}$, $Y_B = -21.43$. If the elevation of point A and B are 440m and 450m determine the flying height. Assume focal length = 150mm

Answer] $AB = 100 \text{ m}$ $X_A = 18.35$ $Y_A = -62.4$ $X_B = 106.41 \text{ mm}$ $Y_B = -21.43$

$AB = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}$

$\frac{X_A}{x_A} = \left(\frac{H - h_A}{f} \right)$

$\frac{Y_B}{y_B} = \left(\frac{H - h_B}{f} \right)$

$$AB = \left\{ \left[x_a \left(\frac{H-h_a}{f} \right) - x_b \left(\frac{H-h_b}{f} \right) \right]^2 + \left[y_a \left(\frac{H-h_a}{f} \right) - y_b \left(\frac{H-h_b}{f} \right) \right]^2 \right\}^{1/2}$$

$$100 = \left\{ \left[18.35 \left(\frac{H-440}{150} \right) - 106.41 \left(\frac{H-450}{150} \right) \right]^2 + \left[-62.4 \left(\frac{H-440}{150} \right) + 21.43 \left(\frac{H-450}{150} \right) \right]^2 \right\}^{1/2}$$

$$100 = \left\{ (-1223(H-440) - 709(H-450))^2 + (-416(H-440) + 142(H-450))^2 \right\}^{1/2}$$

$$100 = \left\{ (265.23 - .586H)^2 + (119.14 - .274H)^2 \right\}^{1/2}$$

~~100~~

$$H = 605.05 \text{ m}$$

Note: This is one of the method available to compute the flying height of vertical photograph. However it can also be computed with the help of focal length and scale of photograph

Notes:

$$\frac{S}{H-h} = \frac{f}{H_1} \Rightarrow H' = \frac{f}{S}$$

Flying Height of the aircraft is decided on the basis of Contour Interval on the topographic map flying height is given by

$$\text{Flying Ht} = [\text{C-factor}] \cdot \text{Contour Interval}$$

C-factor = 500-1500, depending upon the condition of map compilation

PHOTOGRAPHIC OVERLAP

In ~~one~~ Aerial Survey of vertical photograph proper time interval is kept/given b/w two photograph to get the desired overlap of the photograph.

- An overlap is the common area covered b/w the two photograph
- It can be in both longitudinal and lateral direction the former is termed as overlap / longitudinal overlap / End lap forward and lateral is termed as side lap / lateral.
- In general longitudinal overlap is 60% and side lap is (25-35%) - (30%).

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Alternate Photographs have an end overlap of 20%.

- overlap are Required, as distortion of photograph is caused by Relief at the End.
- Side overlaps are required to cover the gaps which are left uncovered in single flight path.

CALCULATE THE REQUIRED NUMBER OF PHOTOGRAPH

- To Calculate no of Photograph to require to cover the given area on ground can be determined as follows.

(A) [When Ground Area is known & A]

$$\text{No of Photograph Required} = \frac{A}{a}, \quad a = \text{Area Covered by one Photograph}$$

$$A = L \times B$$

$$a = l' \times b'$$

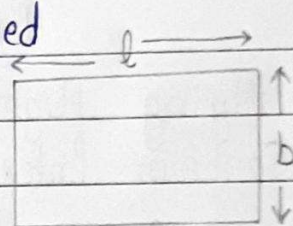
A = Ground area to be covered

l' = length Covered by one photograph on ground

b' = width

S = Scale of photograph

P_L = longitudinal overlap, P_S = Side lap.



$$a = \frac{(1 - P_L) l}{S} \times \frac{(1 - P_S) b}{S}$$

$$\text{No of Photograph} = \frac{A S^2}{(1 - P_L) \times l (1 - P_S) b}$$

~~Here~~ Note: Here only lap area is considered to Account for area covered.

(B) When length and width on ground is known.

N_1 = No of photograph along the length of flight

$$= \left[\left\{ \frac{\frac{L}{S}}{(1 - P_L)} \right\} + 1 \right]$$

Note: 1 additional photograph is taken to cover the unlapped area at the end.

$$N_2 = \frac{B}{(1-P_s)\frac{b}{S}} + 1$$

$N_2 = \text{No of Photograph 1 to direction of flying}$

Total No of Photograph Required $N = N_1 \times N_2$

$$N = \left\{ \frac{L}{(1-P_L)\frac{l}{S}} + 1 \right\} \times \left\{ \frac{B}{(1-P_s)\frac{b}{S}} \right\}$$

Question: Determine the number of photograph required to cover an area of 1000m^2 if the scale of photograph is $1:1000$ and the photograph format is $230\text{mm} \times 230\text{mm}$. Take longitudinal overlaps as 60% and side overlap as 30% .

Answer)
$$N = \frac{AS^2}{(1-P_L) \times l \times (1-P_s) \times b} = \frac{1000 \times [1/1000]^2}{(1-0.6) \times 230 \times (1-0.3) \times 230} = 676$$

Question: Determine the no of photograph required to cover an area $40\text{km} \times 25\text{km}$. If the scale of photograph is $1\text{m}/1000$ and the format is $230 \times 230\text{mm}^2$, $P_L = 60\%$.

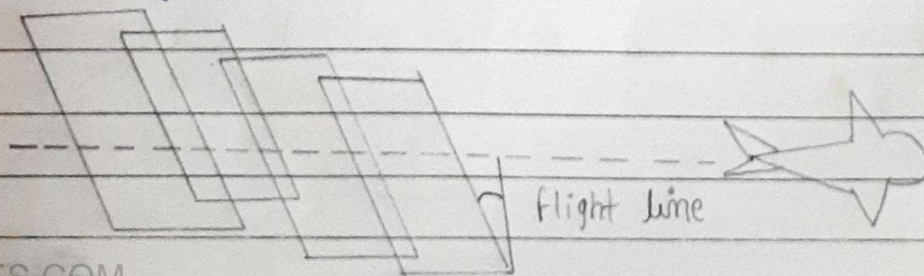
Answer) $P_s = 30$

$$N = \left[\frac{L}{(1-P_L)\frac{l}{S}} + 1 \right] \times \left[\frac{B}{(1-P_s)\frac{b}{S}} \right] \Rightarrow \left[\frac{40 \times 10^3}{(1-0.6) \times 230 \times 1/1000} \right] \times \left[\frac{25 \times 10^3}{(1-0.3) \times 230 \times 1/1000} \right]$$

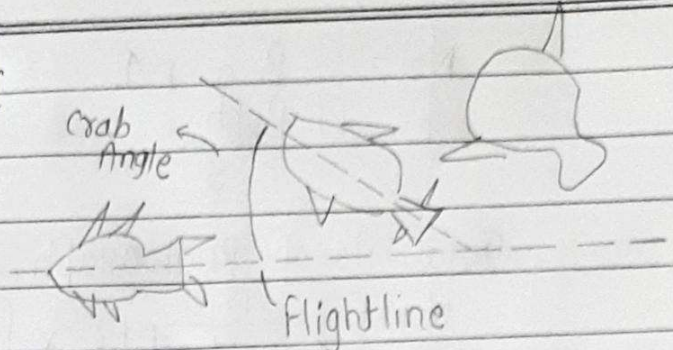
$$= 444.7 + 155.27 = 599.97$$

CRAB AND DRIFT

Crab of the photograph is the angle formed by edge of photograph with flight line



It occurs when aircraft is not oriented with flight line i.e. Photo Edges are not Parallel to flight line



It usually occurs aircraft

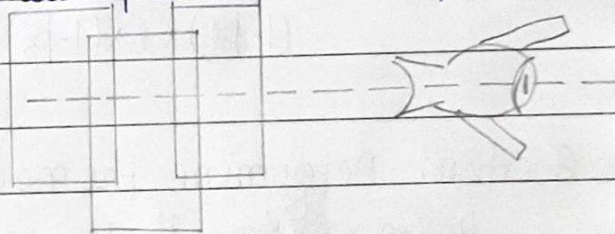
Compensate for a cross wind and orients into the wind with flight line.

It must be eliminated as it reduces the effective coverage over the ground.

- To correct crab, Rotate the Camera about vertical axis.

DRIFT: It is the lateral displacement of photograph

- The photograph does not stay on its predetermined path due to effect of wind



It can be corrected crab is introduced while avoiding the drift

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PARALLAX / X-PARALLAX / STEREOSCOPIC PARALLAX

The change in position of an image from one photograph to another photograph is termed as Parallax. There is always Parallax images which appear over successive overlappings photographs.

